Kentucky Academic Standards
Mathematics

INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states and other parts of the world

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
    - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;
    - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
    - Become self-sufficient individuals;

    - Become responsible members of a family, work group or community as well as an effective participant in community service;
    - Think and solve problems in school situations and in a variety of situations they will encounter in life;
    - Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
    - Express their creative talents and interests in visual arts, music, dance, and dramatic arts.
- Increase student attendance rates.
- Reduce dropout and retention rates.
- Reduce physical and mental health barriers to learning.
- Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> *(Model Curriculum Framework,* page 19*)*

<u>Legal Basis</u>

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

<u>704 KAR 8:040</u> Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;
- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and

- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.

- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

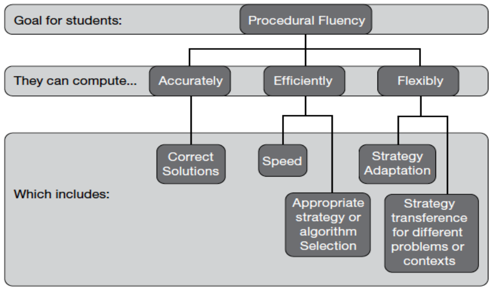
Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).
- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)
- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.



Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.

- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

The Modeling Process

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

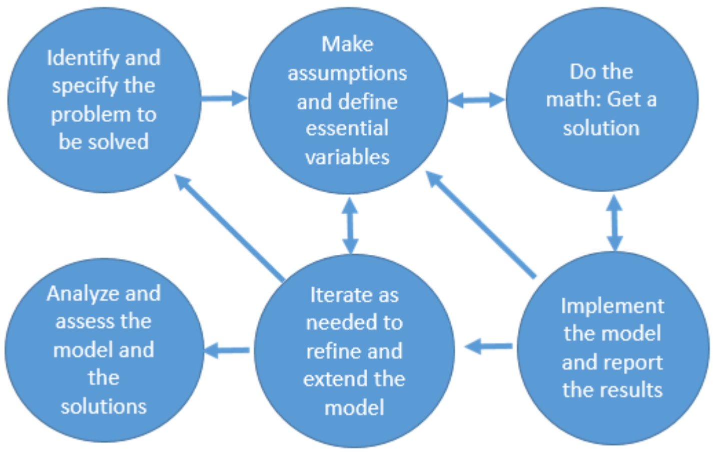
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

<u>The Kentucky Academic Standards (KAS) are Standards, not Curriculum</u>

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate "future-oriented" thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

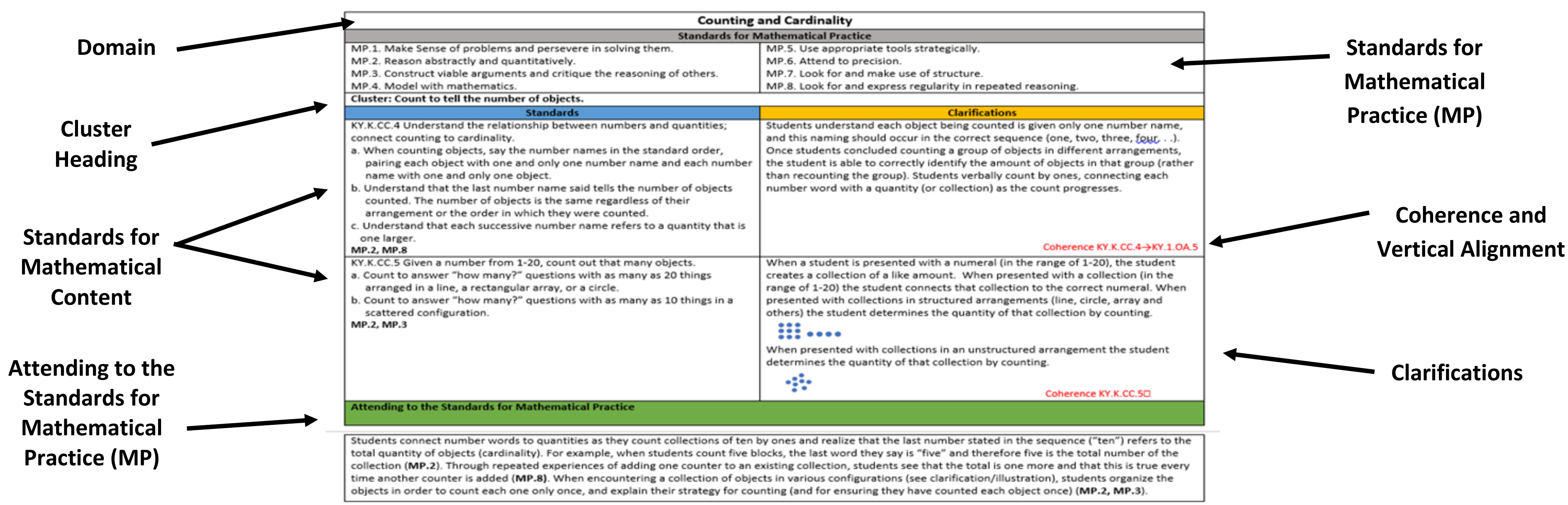
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

Standards for Mathematical Content define what students should understand and be able to do.

Standards for Mathematical Practice define how students engage in mathematical thinking.

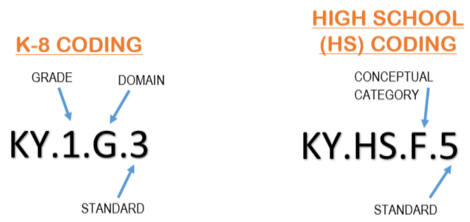
The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.



Counting and Cardinality	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Count to tell the number of objects.	
Standards	**Clarifications**
KY.K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **MP.2, MP.8**	Students understand each object being counted is given only one number name, and this naming should occur in the correct sequence (one, two, three, four, . .). Once students concluded counting a group of objects in different arrangements, the student is able to correctly identify the amount of objects in that group (rather than recounting the group). Students verbally count by ones, connecting each number word with a quantity (or collection) as the count progresses. Coherence KY.K.CC.4→KY.1.OA.5
KY.K.CC.5 Given a number from 1-20, count out that many objects. a. Count to answer "how many?" questions with as many as 20 things arranged in a line, a rectangular array, or a circle. b. Count to answer "how many?" questions with as many as 10 things in a scattered configuration. **MP.2, MP.3**	When a student is presented with a numeral (in the range of 1-20), the student creates a collection of a like amount. When presented with a collection (in the range of 1-20) the student connects that collection to the correct numeral. When presented with collections in structured arrangements (line, circle, array and others) the student determines the quantity of that collection by counting. When presented with collections in an unstructured arrangement the student determines the quantity of that collection by counting. Coherence KY.K.CC.5□
Attending to the Standards for Mathematical Practice	

Students connect number words to quantities as they count collections of ten by ones and realize that the last number stated in the sequence ("ten") refers to the total quantity of objects (cardinality). For example, when students count five blocks, the last word they say is "five" and therefore five is the total number of the collection (**MP.2**). Through repeated experiences of adding one counter to an existing collection, students see that the total is one more and that this is true every time another counter is added (**MP.8**). When encountering a collection of objects in various configurations (see clarification/illustration), students organize the objects in order to count each one only once, and explain their strategy for counting (and for ensuring they have counted each object once) (**MP.2, MP.3**).

How to Read the Coding of the Standards



Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council's 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Supplementary Materials to the Standards

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

Appendix A: Tables

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

Appendix B: Writing and Review Teams

Kentucky Academic Standards for Mathematics: Grade 3 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations and identify and explain patterns in arithmetic.	• Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: A range of algorithms may be used.	• Develop understanding of fractions as numbers. Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.	• Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects. • Understand and apply the statistics process. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	• Reason with shapes and their attributes.

In grade 3, instructional time should focus on four critical areas:

1. **In the Operations and Algebraic Thinking domain, students will:**
   - develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays and area models; multiplication is finding an unknown product and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size;
   - use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors; and
   - compare a variety of solution strategies to learn the relationship between multiplication and division.

2. In the Number Sense and Operations—Fractions domain, students will:
   - develop an understanding of fractions, beginning with unit fractions;
   - view fractions in general as being built out of unit fractions and use fractions along with visual fraction models to represent parts of a whole;
   - understand that the size of a fractional part is relative to the size of the whole. Use fractions to represent numbers equal to, less than and greater than one; and
   - solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. In the Measurement and Data domain, students will:

- recognize area as an attribute of two-dimensional regions;
- measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area; and
- understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

4. In the Geometry domain, students will:

- compare and classify shapes by their sides and angles; and
- relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Note: Multiplication, division and fractions are the most important developments in grade 3.

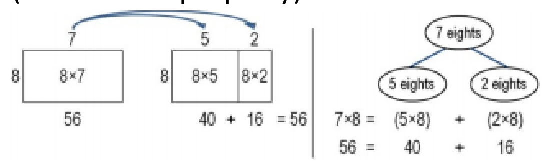
Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and solve problems involving multiplication and division.	
Standards	**Clarifications**
KY.3.OA.1 Interpret and demonstrate products of whole numbers. **MP.2**, **MP.5**	Students use models for multiplication situations. For example, students interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. Coherence KY.2.OA.4→KY.3.OA.1→KY.4.OA.1
KY.3.OA.2 Interpret and demonstrate whole-number quotients of whole numbers, where objects are partitioned into equal shares. **MP.2**, **MP.5**	Students use models for division situations. For example, students interpret 56 ÷ 8 as the number of 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 object each. Coherence KY.3.OA.1→KY.3.OA.2→KY.5.NF.3
KY.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, by using drawings and equations with a symbol for the unknown number to represent the problem. **MP.1**, **MP.4**	Students flexibly model or represent multiplication and division situations or context problems (involving products and quotients up to 100). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. **See Table 2 in Appendix A.** Coherence KY.3.OA.3→KY.4.OA.2
KY.3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **MP.6**, **MP.7**	Students determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = □ ÷ 3, 6 x 6 = ?. Coherence KY.3.OA.4→KY.4.MD.3
Attending to the Standards for Mathematical Practice	
Students recognize the numbers and symbols in an equation such as 5 x 8 = 40 are related to a context using groups or arrays (**MP.2**). For example, a student analyzes this equation and tells a story about walking 8 blocks round-trip to and from school each day, connecting to the equation by saying: 5 days x 8 blocks each day is 40 total blocks walked. To represent the problem, they show 5 jumps of 8 on an open number line or show five 8-unit long Cuisenaire Rods (**MP.5**). When reading story situations, students seek to make sense of the story and its quantities (**MP.1**). They do not just lift numbers out or use keywords. To help make sense of the problem, students decide to write an equation or use a number line. In other words they 'mathematize' the situation (**MP.4**). In missing value problems, students attend to what value is unknown and what operation is represented (**MP. 6**) and use this information to determine what value will result in both sides of the equations being equal (**MP.7**).	

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

Standards	Clarifications
KY.3.OA.5 Apply properties of operations as strategies to multiply and divide. **MP.3**, **MP.4**	Students need not use formal terms for these properties. If 6 x 4 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5+2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property).  KY.4.NBT.5 Coherence KY.3.OA.5→KY.4.NBT.6
KY.3.OA.6 Understand division as an unknown-factor problem. **MP.2**	Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. Coherence KY.3.OA.6→KY.4.NBT.6

Attending to the Standards for Mathematical Practice

Students use strategies beyond skip counting to solve multiplication problems. They decide how to use known facts to solve facts like 6 x 9. Students use strategies like Adding a Group, thinking 5 groups of 9 (45) plus one more group (54) and Subtracting a Group, thinking 9 x 6 and reasoning 10 groups of 6 (60) minus one group of 6 (54) (**MP.7**). Students explain their selected reasoning strategy and listen and critique other students' strategies, considering which strategies make sense and are efficient (**MP.3**). Students think about 84 ÷ 4 as, "How many sets of 4 can be made from 84 items?" or "How many in a group, if there 84 items and 4 groups?" and use this relationship to solve the problem (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Multiply and divide within 100.	
Standards	**Clarifications**
KY.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. **MP.2**, **MP.8**	Students determine multiplication and division strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Knowing 8 x 5 = 40, one knows 40 ÷ 5 = 8. Note: Reaching fluency is an ongoing process that will take much of the year. Coherence KY.3.OA.7→KY.4.OA.4
Attending to the Standards for Mathematical Practice	
By studying patterns and relationships in multiplication facts, students develop fluency for multiplication facts (**MP.8**). For example, students notice 4 x 6 is equivalent to 2 x 2 x 6 (doubling strategy). They know 9 facts can be found by thinking of the other factor x 10 and subtracting one group. For example, recognizing 9 x 8 is equivalent to 10 x 8 – 8. For each fact, the student thinks, "What reasoning strategy can I use that is more efficient than skip counting?" (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

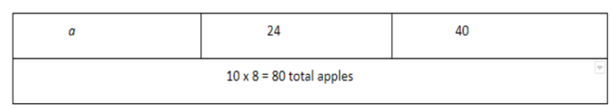
Cluster: Solve problems involving the four operations and identify and explain patterns in arithmetic.

Standards	Clarifications
KY.3.OA.8 Use various strategies to solve two-step word problems using the four operations (involving only whole numbers with whole number answers). Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **MP.1**, **MP.4**	Students solve problems using models, pictures, words and numbers. Students explain how they solved the problem using accurate mathematical vocabulary and why their answer makes sense. Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.2.OA.1→KY.3.OA.8→ KY.4.OA.3
KY.3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. **MP.3**, **MP.8**	Students observe 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends. Coherence KY.2.OA.3→KY.3.OA.9→ KY.4.OA.5

Attending to the Standards for Mathematical Practice

Given a non-straightforward story situation about gathering apples and sharing them among 8 families, students decide on ways to make sense of the problem (**MP.1**). One student decides to use a bar diagram to make sense of the situation and then use the bar diagram to write equations and solve the problem (**MP.4**).

Maggie was picking apples from her three apple trees. She picked some from the first tree and realized she should count the rest of what she was picking. She picked 24 apples from the second tree and 40 apples from the third tree. She had enough apples to give 10 to each of eight families. How many apples did she pick from the first tree?



$a + 24 + 40$ = total apples and 10×8 = total apples. There are 80 apples total.

$a + 64 = 80$

$a = 16$

Another student thinks of the situation differently and decides to figure out how many apples each family has from the known apples (**MP.1**). Other students use counters to model the problem and/or use trial and error. If their first approach doesn't work, students persevere by trying another strategy (**MP.1**). In each case, students check to see if the answer of 16 apples makes sense.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: A range of algorithms may be used.

Standards	Clarifications
KY.3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100. **MP.7**	On a number line, students determine 178 rounded to nearest 10 is 180.  Coherence KY.2.NBT.1→KY.3.NBT.1→ KY.4.NBT.3
KY.3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and/or the relationship between addition and subtraction. **MP.2**, **MP.3**	Students determine addition and subtraction strategies efficiently, accurately, flexibly and appropriately. Being fluent means students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches and they are able to produce accurate answers efficiently. Note: Reaching fluency is an ongoing process that will take much of the year. KY.2.NBT.5 Coherence KY.2.NBT.7→KY.3.NBT.2→ KY.4.NBT4
KY.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range of 10–90 using strategies based on place value and properties of operations. **MP.7**, **MP.8**	To solve 8 x 60, students interpret this as 8 groups of 6 tens, which is 480. KY.3.OA.5 Coherence KY.2.NBT.1→KY.3.NBT.3→ KY.4.NBT.5

Attending to the Standards for Mathematical Practice

Students look at the numbers in a problem and consider which strategy they will use to solve the given problem (**MP.2**). For example, for the problem 405 - 381, a student notices these values are close to each other, so rather than take away 381, they find the difference. They count up to 400 (19) and add on 5 more to equal 24. For the problem 425 - 98, the student notices 98 is close to 100, so chooses to take away 100 and add 2 more back on to equal 327. Students share the strategy they used, why it works and why they chose it (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations-Fractions

Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Develop understanding of fractions as numbers. Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>
KY.3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

MP.2, MP.7
</td><td>
Students name parts of the whole using fractions and explain the fraction is made up of unit fractions. Students describe the numerator and the denominator using pictures, numbers and words.

$\frac{4}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Coherence KY.2.G.3→KY.3.NF.1→ KY.4.NF.3
</td></tr>
<tr><td>
KY.3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line.

a. Represent a fraction $\frac{1}{b}$ (unit fraction) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.

- Recognize each part has size $\frac{1}{b}$.
- a unit fraction, $\frac{1}{b}$ is located $\frac{1}{b}$ of a whole unit from 0 on the number line.

b. Represent a non-unit fraction $\frac{a}{b}$ on a number line by marking off lengths of $\frac{1}{b}$ (unit fractions) from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the non-unit fraction $\frac{a}{b}$ on the number line.

MP.4
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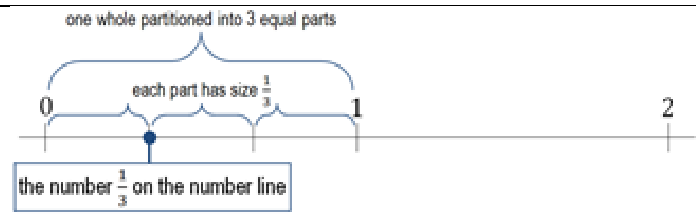
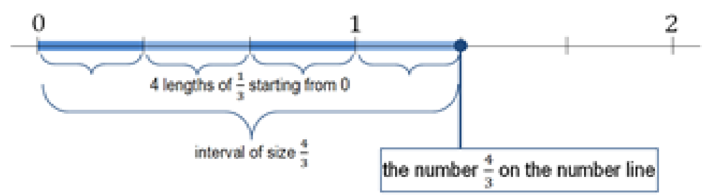




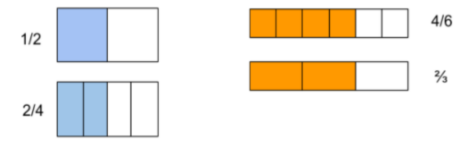
Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Coherence KY.2.MD.6→KY.3.NF.2 →KY.4.NF.3
</td></tr>
</table>

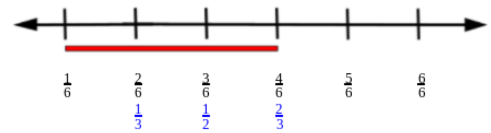
KY.3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

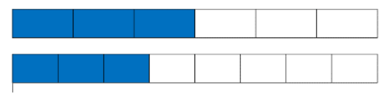
a. Understand two fractions as equivalent (equal) if they are the same size, or same point on a number line.
b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent through writing or drawing.
c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

MP.2, **MP.3**



When working with the same whole, students can see that ½ = 2/4, and 4/6 = ⅔.





$\frac{3}{6}$ is greater than $\frac{3}{8}$ or $\frac{3}{6} > \frac{3}{8}$

Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

KY.4.NF.1

Coherence KY.3.NF.3→ KY.4.NF.5

Attending to the Standards for Mathematical Practice

Students use the number line to reason about the relative size of a fraction (**MP.4**). They locate $\frac{5}{6}$ on a number line by accurately partitioning the line into 6 equal-length segments. They explain that $\frac{5}{6}$ means five segments that are each one-sixth of a unit in length, for example counting, "One-sixth, two-sixths, three-sixths, four-sixths, five-sixths." (**MP.7**). As they partition the line in other ways, they recognize three-sixths is half of the distance to 1 whole, as is $\frac{2}{4}$, $\frac{1}{2}$, and $\frac{4}{8}$, and reason these fractions are equivalent (**MP.2**). Similarly, they can generate other illustrations or justifications to explain why two fractions are equivalent or not (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.	
Standards	**Clarifications**
KY.3.MD.1 Tell and write time to the nearest minute and measure elapsed time intervals in minutes. Solve word problems involving addition and subtraction of time intervals within and across the hour in minutes. **MP.4**, **MP.6**, **MP.1**, **MP.4**	Students solve elapsed time problems using strategies and tools such as clock models and number lines (seeing a clock as a number line). Coherence KY.2.MD.7→KY.3.MD.1→ KY.4.MD.2
KY.3.MD.2 Measure and solve problems involving mass and liquid volume. a. Measure and estimate masses and liquid volumes of objects using standard units of grams (g), kilograms (kg) and liters (L). b. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. **MP.1**, **MP.6**	a. Students have multiple opportunities to weigh classroom objects and fill containers to help them develop a basic understanding of the size and weight of a liter, a gram and a kilogram. b. **See Table 2 in Appendix A.** Coherence KY.2.MD.5→KY.3.MD.2→KY.4.MD.1
Attending to the Standards for Mathematical Practice	
Students solve story situations using a model to support their reasoning (**MP.4**). For example, a student solves a task such as: you try to run for 15 minutes without stopping. When you look at the clock, the time is 2:52. What time will it say when you have reached 15 minutes? On an open number line, they show a jump from 2:52 to 3:00 as 8 minutes and then jump 7 minutes more to 3:07. Students estimate and then measure objects using standard units. For example, how many grams might balance with a selected item (**MP.6**)?	

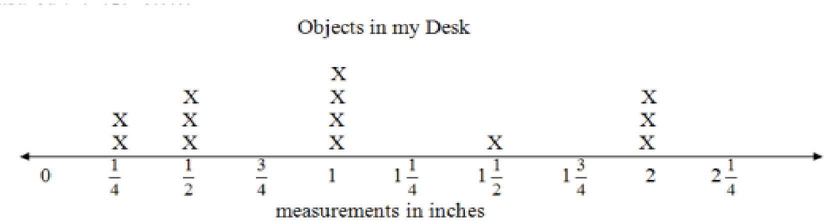
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand and apply the statistics process.

Standards	Clarifications
KY.3.MD.3 Investigate questions involving categorical data. a. Identify a statistical question focused on categorical data and gather data; b. Create a scaled pictograph and a scaled bar graph to represent a data set (using technology or by hand); c. Make observations from the graph about the question posed, including "how many more" and "how many less" questions. **MP.3**, **MP.5**, **MP.6**	Students select a question of interest (how many pets does each classmate have), gather data and create a bar graph (each square in the bar graph might represent 2 pets). Coherence KY.2.MD.10→KY.3.MD.3
KY.3.MD.4 Investigate questions involving numerical data. a. Identify a statistical question focused on numerical data; b. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. c. Show the data by making a dot plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters. d. Make observations from the graph about the question posed, including questions about the shape of the data and compare responses. **MP.1**, **MP.3**, **MP.6**	Students measure objects in their desk to the nearest ½ or ¼ of an inch, display data collected on a dot plot and analyze the data.  Coherence KY.2.MD.9→KY.3.MD.4→KY.4.MD.4

Attending to the Standards for Mathematical Practice

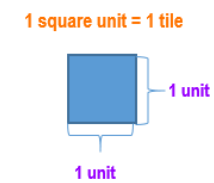
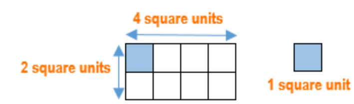
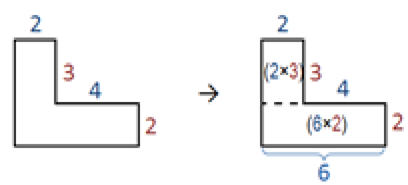
Students understand the purpose of creating a graph is to make sense of data related to a question (**MP.1**). They look at the data they have collected and decide on how to set up a graph to best communicate the data (**MP.6**). Students determine if the scale on a dot plot should be in whole numbers, halves or fourths, based on the data gathered. For example, if they measured the length of each person's pencil to the nearest fourth inch, the related dot plot would be created using fourths (**MP.6**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understanding concepts of area and relate area to multiplication and to addition.

Standards	Clarifications
KY.3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. **MP.5**	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area.  A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.  Coherence KY.3.MD.5→KY.5.MD.3
KY.3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft. and improvised units). **MP.5**, **MP.6**	Students use grid paper of varying square units to count the number of unit squares in a figure. Coherence KY.2.G.2→KY.3.MD.6→KY.5.MD.4
KY.3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$	d.  Coherence KY.3.MD.7→KY.4.MD.3→ KY.5.MD.5

and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of figures that can be decomposed into non-overlapping rectangles by adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **MP.1**, **MP.8**	
Attending to the Standards for Mathematical Practice	
Students use 1 inch color tiles to cover a rectangle, understanding that color tile as a square inch (**MP.5**). As students place the tiles in repeated rows to fill the rectangle, they notice each row has the same number of tiles and the number of tiles that will fill a rectangle can be written as [number of tiles in one row] x [number of rows] (**MP.8**). They solve story problems that sometimes have the area as the unknown and sometimes have the number of rows or columns as the unknown and use their knowledge of area to solve the problem (**MP.1**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Standards

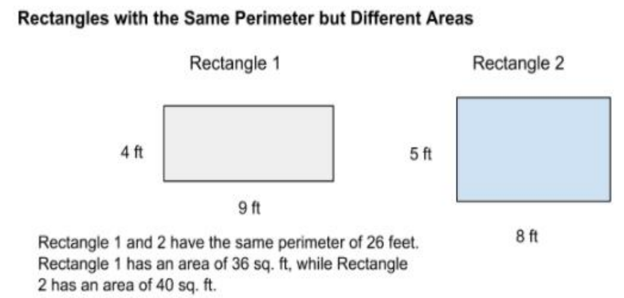
KY.3.MD.8 Solve real world and mathematical problems involving perimeters of polygons.

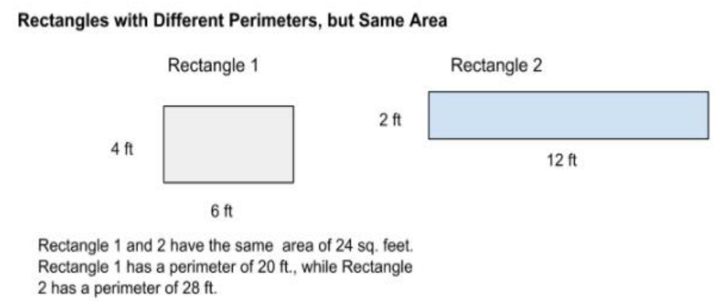
a. Find the perimeter given the side lengths of a polygon.
b. Find an unknown side length, given the perimeter and some lengths.
c. Draw rectangles with the same perimeter and different areas or with the same area and different perimeters.

MP.1, **MP.4**

Clarifications

c.





Coherence KY.3.MD.8→KY.4.MD.3

Attending to the Standards for Mathematical Practice

Students recognize perimeter is a measure of length and see perimeters of polygons as a collection of side lengths added together to form the perimeter (**MP.1**). Therefore, they see if a side length is missing, it is like a missing addend problem and write an equation or draw a bar diagram to solve for the missing value (**MP.4**). Students recognize they can use a given perimeter (such as 16 inches) and form different rectangles (such as 4 x 4, 3 x 5, 2 x 6, 1 x 7) and that these rectangles have different areas (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.3.G.1 Classify polygons by attributes. a. Recognize and classify polygons based on the number of sides and vertices (triangles, quadrilaterals, pentagons and hexagons). b. Recognize and classify quadrilaterals (rectangles, squares, parallelograms, rhombuses, trapezoids) by side lengths and understanding shapes in different categories may share attributes and the shared attributes can define a larger category. c. Identify shapes that do not belong to a given category or subcategory. **MP.6**, **MP.7**	Students describe, analyze and compare properties of two–dimensional shapes. Coherence KY.2.G.1→KY.3.G.1→KY.4.G.2
KY.3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. **MP.2**, **M.5**	Partitioned parts should be halves, thirds, fourths, sixths, eighths. Students partition a shape into 6 parts with equal areas and describe the area of each part as $\frac{1}{6}$ of the area of the shape. KY.3.NF.1 Coherence KY.2.G.3→KY.3.G.2
Attending to the Standards for Mathematical Practice	
Students describe attributes they notice for a particular type of quadrilateral, focusing on side lengths and angles (**MP.6**). They explain what different types of quadrilaterals have in common and can distinguish between what are defining attributes (such as having four sides) and what are not defining (such as its size or color) (**MP.3**). Students use a variety of tools and drawings to show fractional parts (**MP.5**) and they reason if a shape is partitioned into four equal-sized parts (even if they are not the same shape), each part represents one-fourth of the whole shape (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1
Common Addition and Subtraction Situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	Group Size Unknown	Number of Groups Unknown
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3
Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\, x\, (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c then a > c
If a > b, then b < a
If a > b, then –a < -b
If a > b, then a ± c > b ± c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a ÷ c > b ÷ c
If a > b and c < 0, then a ÷ c < b ÷ c

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★