Kentucky Academic Standards
Mathematics

INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states and other parts of the world

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
    - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;
    - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
    - Become self-sufficient individuals;

    - Become responsible members of a family, work group or community as well as an effective participant in community service;
    - Think and solve problems in school situations and in a variety of situations they will encounter in life;
    - Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
    - Express their creative talents and interests in visual arts, music, dance, and dramatic arts.
- Increase student attendance rates.
- Reduce dropout and retention rates.
- Reduce physical and mental health barriers to learning.
- Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> *(Model Curriculum Framework,* page 19*)*

Legal Basis

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

704 KAR 8:040 Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;
- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and

- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.

- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

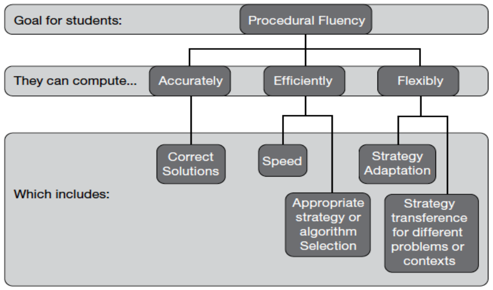
Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).
- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)
- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.



Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.

- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

The Modeling Process

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

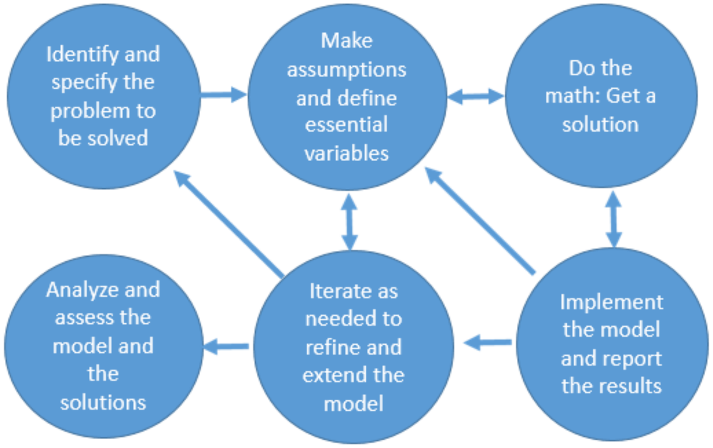
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

The Kentucky Academic Standards (KAS) are Standards, not Curriculum

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate “future-oriented” thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

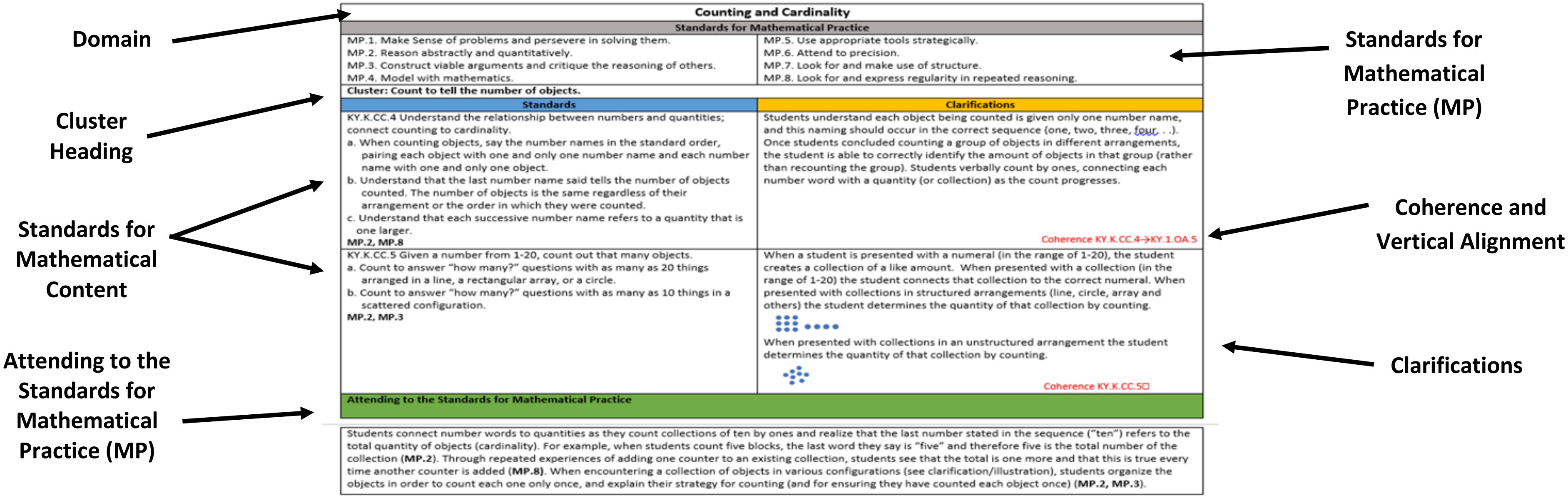
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

Standards for Mathematical Content define what students should understand and be able to do.

Standards for Mathematical Practice define how students engage in mathematical thinking.

> *The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.*



Counting and Cardinality

Standards for Mathematical Practice

MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

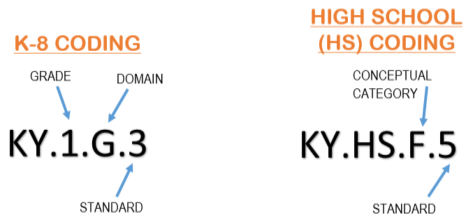
Cluster: Count to tell the number of objects.

Standards	**Clarifications**
KY.K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **MP.2, MP.8**	Students understand each object being counted is given only one number name, and this naming should occur in the correct sequence (one, two, three, four, . . .). Once students concluded counting a group of objects in different arrangements, the student is able to correctly identify the amount of objects in that group (rather than recounting the group). Students verbally count by ones, connecting each number word with a quantity (or collection) as the count progresses. Coherence KY.K.CC.4→KY.1.OA.5
KY.K.CC.5 Given a number from 1-20, count out that many objects. a. Count to answer "how many?" questions with as many as 20 things arranged in a line, a rectangular array, or a circle. b. Count to answer "how many?" questions with as many as 10 things in a scattered configuration. **MP.2, MP.3**	When a student is presented with a numeral (in the range of 1-20), the student creates a collection of a like amount. When presented with a collection (in the range of 1-20) the student connects that collection to the correct numeral. When presented with collections in structured arrangements (line, circle, array and others) the student determines the quantity of that collection by counting. When presented with collections in an unstructured arrangement the student determines the quantity of that collection by counting. Coherence KY.K.CC.5☐

Attending to the Standards for Mathematical Practice

Students connect number words to quantities as they count collections of ten by ones and realize that the last number stated in the sequence ("ten") refers to the total quantity of objects (cardinality). For example, when students count five blocks, the last word they say is "five" and therefore five is the total number of the collection (**MP.2**). Through repeated experiences of adding one counter to an existing collection, students see that the total is one more and that this is true every time another counter is added (**MP.8**). When encountering a collection of objects in various configurations (see clarification/illustration), students organize the objects in order to count each one only once, and explain their strategy for counting (and for ensuring they have counted each object once) (**MP.2, MP.3**).

How to Read the Coding of the Standards



Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Supplementary Materials to the Standards

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

Appendix A: Tables

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

Appendix B: Writing and Review Teams

Kentucky Academic Standards for Mathematics: Grade 4 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Use the four operations with whole numbers to solve problems. • Gain familiarity with fractions and multiples. • Generate and analyze patterns.	• Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.	• Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions and compare decimal fractions.	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Understand and apply the statistics process. • Geometric measurement: understand concepts of angle and angle measurements.	• Draw and identify lines and angles and classify shapes by properties of their lines and angles.

In grade 4, instructional time should focus on three critical areas:

1. **In the Number and Operations in Base Ten domain, students will:**
    - generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place;
    - apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value and properties of operations, in particular the distributive property, as they develop, discuss and use efficient, accurate and generalizable methods to compute products of multi-digit whole numbers;
    - determine and accurately apply appropriate methods to estimate or mentally calculate products;
    - develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems;
    - apply their understanding of models for division, place value, properties of operations and the relationship of division to multiplication as they develop, discuss and use efficient, accurate and generalizable procedures to find quotients involving multi-digit dividends;
    - select and accurately apply appropriate methods to estimate and mentally calculate quotients and interpret remainders based upon the context.

2. **In the Numbers and Operations--Fractions domain, students will:**
    - create an understanding of fraction equivalence and operations with fractions;
    - recognize that two different fractions can be equal and they develop methods for generating and recognizing equivalent fractions;
    - extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions; decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. **In the Geometry domain, students will:**

- describe, analyze, compare and classify two-dimensional shapes;
- strengthen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry through building, drawing and analyzing two-dimensional shapes.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use the four operations with whole numbers to solve problems.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.4.OA.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.
MP.2, MP.4</td><td>Students interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.
Coherence KY.3.OA.1→ KY.4.OA.1→KY.5.NF.5</td></tr>
<tr><td>KY.4.OA.2 Multiply or divide to solve word problems involving multiplicative comparisons by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
MP.1, MP.2, MP.3</td><td>Students solve multiplicative comparison problems using drawings and equations to determine situations like the ones below (Table 2 in Appendix A) on which quantity is being multiplied and which factor is telling how many times.
<table>
<tr><th colspan="3">Common Comparison Problems for Multiplication and Division</th></tr>
<tr><td>Unknown product</td><td>Group size unknown</td><td>Number of groups unknown</td></tr>
<tr><td>A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?

Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</td><td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?
Measurement example: A rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?</td><td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue?
Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td></tr>
<tr><td>$a \times b = ?$</td><td>$a \times ? = p$ and $p \div a = ?$</td><td>$? \times b = p$ and $p \div b = ?$</td></tr>
</table>
Coherence KY.3.OA.3→ KY.4.OA.2→KY.5.NF.3</td></tr>
</table>

KY.4.OA.3 Solve multistep problems. a. Perform operations in the conventional order when there are no parentheses to specify a particular order. b. Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computations and estimation strategies including rounding. **MP.1**, **MP.4**	a. Students use their knowledge of order of operations even when there are no parentheses or brackets. $31 + 3 \times 8 - 20 =$ b. For example, Mr. May's grade four class is collecting canned goods for a food drive. Their goal is to bring in 50 cans of food by Friday. So far, the students have brought in 10 on Monday and Tuesday, 14 cans on Wednesday and 13 on Thursday. How many more cans will the class need to bring in to reach their goal? $50 = 2 \times 10 + 14 + 13 + \boldsymbol{c}$ $50 = 20 + 14 + 13 + \boldsymbol{c}$ $50 = 47 + \boldsymbol{c}$ $\boldsymbol{3 = c}$ Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.3.OA.8→ KY.4.OA.3→KY.7.NS.3

Attending to the Standards for Mathematical Practice

Students recognize a number represents a specific quantity and connects the quantity to written symbols and creates a logical representation of the problem considering both the appropriate units involved and the meaning of quantities (**MP2**). In an equation such as 35= 5 x 7, students identify and verbalize which quantity is being multiplied and which number tells how many times, saying, "Sally is five years old. Her mom is seven times older. How old is Sally's Mom?"

Students discover a pattern or structure (**MP.7**). For example, a student distinguishes an additive comparison by identifying this type of question asks, " How many more?" and a multiplicative comparison focuses on comparing two quantities by asking," How many times as much?" or "How many times as many?" Students solve contextual problems using models and equations using a symbol to represent the unknown (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Cluster: Gain familiarity with factors and multiples.

Standards	Clarifications
KY.4.OA.4 Find factors and multiples of numbers in the range 1-100. a. Find all factor pairs for a given whole number. b. Recognize that a whole number is a multiple of each of its factors. c. Determine whether a given whole number is a multiple of a given one-digit number. d. Determine whether a given whole number is prime or composite. **MP.5**, **MP.7**	Students extend their knowledge of multiplication and division facts by exploring patterns they have found by building conceptual understanding of prime numbers (numbers with exactly two factors) and composite numbers (numbers with more than two factors). Patterns include: • Numbers that end in 0 have 10 as a factor. These are multiples of 10. • Numbers that end in 0 or 5 as a factor. These are multiples of 5. • Even numbers have 2 as a factor. These numbers are multiples of 2. • Numbers that can be halved twice have 4 as a factor. These numbers are multiples of 4. Coherence KY.3.OA.7→ KY.4.OA.4→KY.6.NS.4

Attending to the Standards for Mathematical Practice

Students use the structure and pattern of the counting numbers to find factor pairs, recognizing once they reach a certain point they don't have to keep looking for factors (**MP.7**). Students build arrays with a given area and look for patterns such as numbers of possible arrays to identify if the number is prime or composite. For example, noticing the number 7 has only two possible arrays, 1 x 7 and 7 x 1, therefore, it is prime. The number 4 has more than two rectangular arrays, 1 x 4, 4 x 1 and 2 x 2 and therefore, it is composite.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Cluster: Generate and analyze patterns.

Standards	Clarifications
KY.4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern not explicit in the rule itself. **MP.2**, **MP.3**	For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. Coherence KY.3.OA.9→ KY.4.OA.5→KY.5.OA.3

Attending to the Standards for Mathematical Practice

Students analyze growing patterns and determine rules to describe the pattern (**MP.2**). Students know a pattern is a sequence that repeats the same rule over and over. Students generate their own rules and create an example using that rule, for example, they write 1, 3, 9, 27, 81, 243 for the rule "times 3". Students describe features of the pattern for example, all numbers are odd, or sums of the digits equal 9 and the rule for generating the next number, for example "times 3", as well as critique the reasonableness of features and rules they hear from others (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten

Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Generalize place value understanding for multi-digit whole numbers.

Standards	Clarifications
KY.4.NBT.1 Recognize in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. **MP.7**	Students recognize the relationship of same digits located in different places in a whole number. For example, in the number 435, the digit 5 in the ones place, while the digit 5 in 652 is in the tens place. The five in 652 is ten times greater than the five in 435. Coherence KY.2.NBT.1→ KY.4.NBT.1→KY.5.NBT.1
KY.4.NBT.2 Represent and compare multi-digit whole numbers. a. Read and write multi-digit whole numbers using base-ten numerals, number names and expanded form. b. Compare two multi-digit numbers based on meanings of the digit in each place, using >, =, and < symbols to record the results of comparisons. **MP.2**, **MP.7**	a. Students write numbers in three different forms. For example, 435, four hundred thirty-five, 400 + 30 + 5. b. Students use different forms of the number to help compare. For example, when students are comparing numbers, they determine that 453 is greater than 435 because the 5 is worth 50 in 453, while the tens place only has 3 worth 30 in 435. So 453 > 435. Coherence KY.4.NBT.2→KY.5.NBT.3
KY.4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. **MP.2**, **MP.6**	Students go beyond the application of a procedure when rounding. Students demonstrate a deeper understanding of number sense and place value when they explain and reason about the answers they get when rounding. KY.4.OA.3 Coherence KY.3.NBT.1→ KY.4.NBT.3→KY.5.NBT.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students use precise language, such as “ten times as much as” rather than “ten times more than” as they describe place value relationships (**MP.6**). Students make the conceptual connection between place value and multiplying and dividing by 10, noticing when any digit is multiplied by 10, the place of the digit moves one place to the left and when a digit is divided by 10, it moves to one place to the right. Beyond noticing this pattern, students understand this pattern exists because place value is structured this way (**MP.7**). For example, in solving 35 x 10 = ____, students might place 35 in a place value chart and explain 5 tens is 50, therefore, moving the 5 to the tens place and 30 tens equals 3 hundreds, therefore, moving the 3 to the hundreds place.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

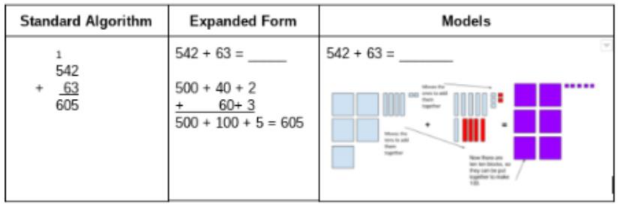
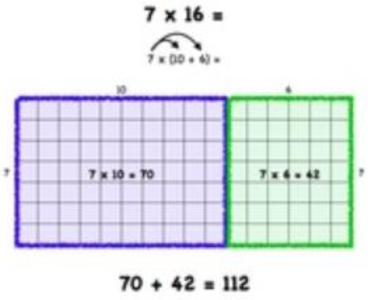
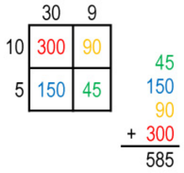
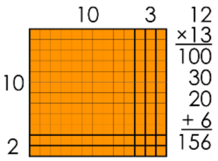
Number and Operations in Base Ten

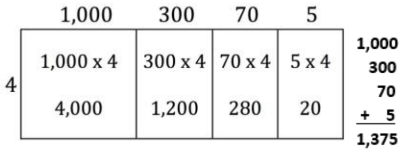
Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standards	Clarifications
KY.4.NBT.4 Fluently add and subtract multi-digit whole numbers using an algorithm. **MP.2**, **MP.8**	Students make connections from previous work with addition and subtraction, using models/representations to develop an efficient algorithm to add and subtract multi-digit numbers. These are types of algorithms/strategies one could possibly use (but not limited to) to solve adding and subtracting multi-digit whole numbers.  Standard Algorithm: 1 / 542 / + 63 / 605 Expanded Form: 542 + 63 = _____ / 500 + 40 + 2 / + 60+ 3 / 500 + 100 + 5 = 605 Models: 542 + 63 = _______ Coherence KY.3.NBT.2→ KY.4.NBT.4→KY.5.NBT.5
KY.4.NBT.5 Multiply whole numbers • Up to four digit number by a one-digit number • Two-digit number by two-digit number Multiply using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. **MP.3**, **MP.4**, **MP.8**	Students use a variety of models (rectangular arrays and area models) and strategies to represent multi-digit factors times a one-digit factor and a two-digit number by a two-digit number. Students also connect their reasoning to a written equation. Some examples include:   

	KY.3.OA.5 Coherence KY.3.NBT.3→ KY.4.NBT.5→KY.5.NBT.5 KY.3.MD.7
KY.4.NBT.6 Divide up to four-digit dividends by one-digit divisors. Find whole number quotients and remainders using • strategies based on place value • the properties of operations • the relationship between multiplication and division Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. **MP.3**, **MP.7**, **MP.8**	Student use a variety of models (rectangular arrays and area models) and strategies to divide up to four-digit dividends by one-digit divisors.  5,500 ÷ 4 = ? Note: By the end of grade 4 students should be able to model, write and explain division by a one-digit divisor. KY.3.OA.5 Coherence KY.3.OA.6→ KY.4.NBT.6→KY.5.NBT.6 KY.3.MD.7

Attending to the Standards for Mathematical Practice

Students select from their repertoire of strategies to solve multi-digit whole number addition or subtraction problems. For example, for the problem 345,402 – 67,087 =□, a student might choose to stack it and subtract using an algorithm. The same student seeing 56,708 – 9,998 = __, might notice how close the subtrahend (second value) is to 10,000 and decide to subtract 10,000 and add 2 onto the answer (**MP.2**). In general, students determine their approach based on the numbers in the problem seeking an efficient strategy. For multiplication and division, students recognize the relationship between area and multiplication and take advantage of rectangular arrays to model multiplication problems (**MP.4**). In creating such models and recording them as equations, students notice repetitive actions in computation and make generalizations to solve other similar problems (**MP.8**). Students explain how and why their selected models and/or algorithms work (**MP.3**).

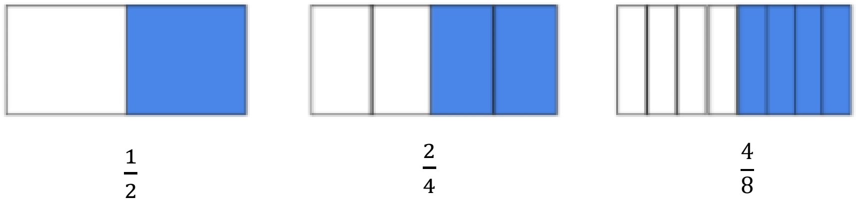
Number and Operations – Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

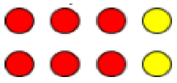
Cluster: Extend understanding of fraction equivalence and ordering.

Standards	Clarifications
KY.4.NF.1 Understand and generate equivalent fractions. a. Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size. b. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$. **MP.4**, **MP.7**, **MP.8**	Students draw fractions and see equivalent fractions.  Coherence KY.3.NF.3→ KY.4.NF.1→KY.5.NF.1
KY.4.NF.2 Compare two fractions with different numerators and different denominators using the symbols <, =, or >. Recognize comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions. **MP.2**, **MP.3**	Students use a variety of representations to compare fractions including concrete models, benchmarks, common denominators and common numerators. Note: Students determine which strategy makes the most sense to them, realizing they use different strategies for different situations. Coherence KY.3.NF.3d→ KY.4.NF.2→KY.5.NF.2

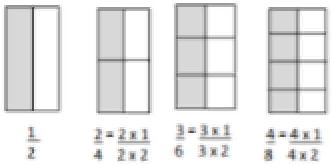
Attending to the Standards for Mathematical Practice

Work in this standard extends the work in grade 3 by using additional denominators (5, 10, 12 and 100). Students use visual models such as area models, number lines, or sets of objects to illustrate how two fractions are equivalent **(MP.4)**

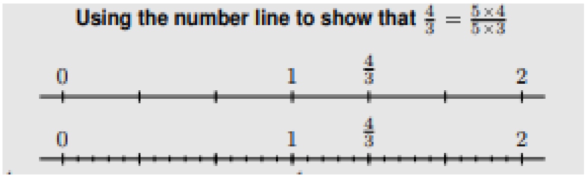
set model $\frac{6}{8} = \frac{3}{4}$



area model $\frac{1}{2} = \frac{4}{8}$



number line $\frac{4}{3} = \frac{20}{15}$



When students are asked to compare two fractions, they do not use a strategy they don't understand, such as the butterfly method, but rather employ reasoning strategies. They first consider whether they can decide which fraction is greater by observation (for example, the fractions have the same numerator or denominator or one fraction is greater than a benchmark and the other is less). If the fractions cannot be compared in this way, students decide whether to find a common denominator or a common numerator and then find the necessary fraction **equivalencies** to compare. For example, to compare $\frac{3}{8}$ and $\frac{5}{12}$, one can see $\frac{5}{12}$ is closer to $\frac{1}{2}$ (only $\frac{1}{12}$ away, while $\frac{3}{8}$ is $\frac{1}{8}$ away) and therefore know that $\frac{5}{12}$ is greater. Another student might not see this relationship, but decide that finding a common numerator is easier (being a basic fact) and multiply $\frac{3}{8}$ by $\frac{5}{5}$ to get $\frac{15}{40}$ and $\frac{5}{12}$ by $\frac{3}{3}$ to get $\frac{15}{36}$. Then recognize and explain that $\frac{15}{36}$ is greater (the pieces are larger) (**MP.2**, **MP.3**).

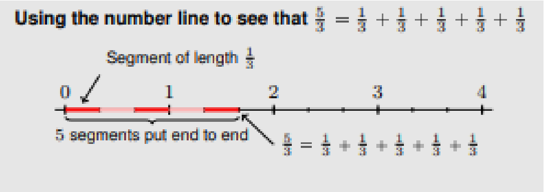
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards	Clarifications
KY.4.NF.3 Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions. c. Add and subtract mixed numbers with like denominators. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. **MP.1, MP.5, MP.7**	b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ OR $\frac{3}{5} = \frac{2}{5} + \frac{1}{5}$ $3\frac{1}{4} = 1 + 1 + 1 + \frac{1}{4}$ OR $3\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$ c/d. Adding and subtracting using visual fraction models and/or equations to represent the problem.  KY.5.NF.1 Coherence KY.3.NF.1→ KY.4.NF.3→KY.5.NF.2
KY.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number. **MP.5, MP.8**	Students refer this standard to *n* groups of a fraction (where *n* is a whole number) for example 3 groups of $\frac{1}{4}$, which can be seen as repeated addition. In grade 5 students will multiply a fraction by a whole number. a. Students use visual fraction models to represent $\frac{7}{5} = 7 \times \frac{1}{5}$ b. Students use the same thinking to see $3 \times \frac{2}{5}$ as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5}$ KY.4.OA.2 Coherence KY.3.NF.1→ KY.4.NF.4→KY.5.NF.4

Attending to the Standards for Mathematical Practice
As students begin to work with fractions greater than unit fractions such as $\frac{2}{3}+\frac{2}{3}=$ ____, they recognize, like whole numbers, they can decompose the non-unit fraction solve problems (Example: $\frac{2}{3}+\frac{2}{3}=\frac{2}{3}+\frac{1}{3}+\frac{1}{3}=1\frac{1}{3}$) (**MP.7**). Students apply this knowledge make sense of word problems and persevere in solving them (**MP.1**). By using tools and situations, students notice a pattern and generalize how to multiply a fraction by a whole number (for example, problems in the form n x $\frac{a}{b}$). For example, they use pattern blocks or Cuisenaire Rods to determine the answer to a set of tasks: 4 x $\frac{1}{2}$, 5 x $\frac{1}{3}$, 6 x $\frac{1}{3}$, 5 x $\frac{2}{3}$, 6 x $\frac{2}{3}$ and notice they multiply to find how many parts and thereby multiplying the whole number by the numerator (**MP.5**, **MP.8**). Note: Following a rote process of "putting a one under the whole number" or other rules not understood work against building understanding of 4.NF.4 and the development of mathematical practices.

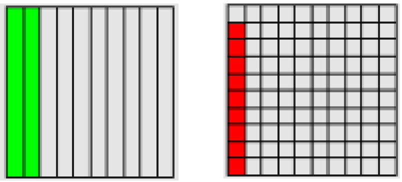
Number and Operations Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand decimal notation for fractions and compare decimal fractions.

Standards	Clarifications
KY.4.NF.5 Convert and add fractions with denominators of 10 and 100. a. Convert a fraction with a denominator of 10 to an equivalent fraction with a denominator of 100. b. Add two fractions with respective denominators 10 and 100. **MP.5**, **MP.7**	For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$ Note: Students who generate equivalent fractions develop strategies for adding fractions with unlike denominators in general. Addition and subtraction with unlike denominators in general is not a requirement at grade 4. Coherence KY.3.NF.3→ KY.4.NF.5→KY.5.NBT.7
KY.4.NF.6 Use decimal notation for fractions with denominators 10 or 100. **MP.4**, **MP.7**	For example, students rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line. Coherence KY.4.NF.6→KY.5.NBT.3
KY.4.NF.7 Compare two decimals to hundredths. a. Compare two decimals to hundredths by reasoning about their size. b. Recognize that comparisons are valid only when the two decimals refer to the same whole. c. Record the results of comparisons with the symbols >, =, or < and justify the conclusions. **MP.2**, **MP.3**, **MP.5**	Students recognize comparisons are valid only when the two decimals refer to the same whole. For example, students use a visual model: seeing 0.2 > 0.09  Coherence KY.4.NF.7→KY.5.NBT.3

Attending to the Standards for Mathematical Practice

Students consider available tools and choose to use base ten blocks, graph paper, place value charts, number lines and other place value models to explore the relationships between fractions with denominators of 10 and denominators of 100 (**MP.5**). By using these tools, students begin to make abstract and quantitative connections to the relationship between fractions with denominators of 10 and 100 (**MP.2**). Through these experiences and work with fraction models, they build the understanding comparisons between fractions and decimals are only valid when the whole is the same for both cases (hundredths or tenths) (**MP.7**)**.** Students use base ten blocks, 10 by 10 geoboards and 10 by 10 grids to illustrate and compare decimal fractions and justify their conclusions (**MP.3**, **MP.5**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.4.MD.1 Know relative size of measurement units (mass, weight, liquid volume, length, time) within one system of units (metric system, U.S. standard system and time).
a. Understand the relationship of measurement units within any given measurement system.
b. Within any given measurement system, express measurements in a larger unit in terms of a smaller unit.
c. Record measurement equivalents in a two-column table.
MP.5, MP.6</td><td>c. Two- column tables may include:
<table><tr><th>kg</th><th>g</th></tr><tr><td>1</td><td>1000</td></tr><tr><td>2</td><td>2000</td></tr><tr><td>3</td><td>3000</td></tr></table><table><tr><th>ft</th><th>in</th></tr><tr><td>1</td><td>12</td></tr><tr><td>2</td><td>24</td></tr><tr><td>3</td><td>36</td></tr></table><table><tr><th>lb</th><th>oz</th></tr><tr><td>1</td><td>16</td></tr><tr><td>2</td><td>32</td></tr><tr><td>3</td><td>48</td></tr></table>Coherence KY.4.MD.1→KY.5.MD.1</td></tr>
<tr><td>KY.4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money.
a. Solve measurement problems involving whole number, simple fractions or decimals.
b. Solve problems that require converting a given measurement from a larger unit to a smaller unit within a common measurement system, such as 2 km = 2,000 m.
c. Visually display measurement quantities using representations such as number lines that feature a measurement scale.
MP.1, MP.4</td><td>Note: grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
Coherence KY.3.MD.2→KY.4.MD.2</td></tr>
<tr><td>KY.4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.
MP.1, MP.3</td><td>Students apply the area and perimeter formulas to real world problems with an unknown factor:
Area = length × width (A = <i>l</i> × w)</td></tr>
</table>

	perimeter = length + width + length + width ($p = l + w + l + w$ OR $p = 2l + 2w$) KY.3.MD.8 Coherence KY.3.MD.7→KY.4.MD.3→KY.5.MD.5

Attending to the Standards for Mathematical Practice

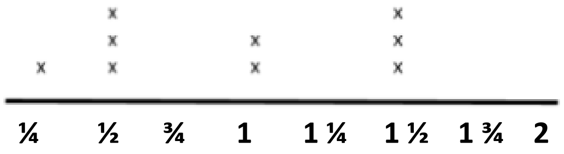
Students know relative sizes of measurement units by actually measuring with the units and establishing a reference to an object. For example, recognizing a centimeter is about the width of their finger (**MP.5**). Students also measure objects using different units within the same system, such as meters and in centimeters (using a meter stick). Record the measurements in a table and notice relationships (**MP.8**). They explain why this pattern is true, arguing each meter has 100 centimeters, so 3 meters will have 300 centimeters and more generally explaining the smaller the unit the more units there will be when measuring the same object (**MP.3**). As students solve problems, they attend to and explain the attribute being measured (length or area), the unit being used to measure and make sense of the problem using drawings, tools, or strategies that make sense to them (**MP.1**, **MP.3**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand and apply the statistics process.

Standards	Clarifications
KY.4.MD.4 Use dot plots to analyze data to a statistical question. a. Identify a statistical question focused on numerical data. b. Make a dot plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). c. Solve problems involving addition and subtraction of fractions by using information presented in dot plots. **MP.1**, **MP.6**	Students create dot plots to show a data set of objects with fractional measurements.  Coherence KY.3.MD.4→KY.4.MD.4→KY.5.MD.2

Attending to the Standards for Mathematical Practice

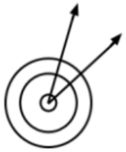
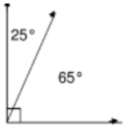
Students recognize a statistical question is one that has variability in the answer and create such a question of interest to them and for which there are numerical responses (**MP.1**). After gathering data on a question of interest, students recognize they have many data points and therefore creating a graph helps to analyze the data. In creating the dot plot, students create a scale from 0 to 1 and label the scale to include intervals of $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ (**MP.6**). As they solve problems related to the graph, they stay focused on the reason they created the graph - to provide insights into the question they first posed, so responses focus on the statistical question posed (**MP.1**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understand concepts of angle and measure angles.

Standards	**Clarifications**
KY.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement. **MP.7**	An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. Angles are measured in reference to a circle with the center at that common point. 
KY.4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **MP.5**, **MP.6**	KY.4.MD.6 Coherence KY.4.MD.5→KY.4.MD.7
KY.4.MD.7 Recognize angle measure as additive. When an angle is into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. **MP.1**, **MP.4**	For example, students use an equation with a symbol for the unknown angle measure.  25° + ⍰ = 90° Coherence KY.4.MD.7→KY.7.G.5

Attending to the Standards for Mathematical Practice

Students explore angle measures using tools (**MP.5**). For example, the white rhombus in a pattern block set or a cardboard cut-out is used as a 'unit' angle (a non-standard unit). Students use this tool to measure the size of other angles, noticing that angle measures are additive (**MP.1**). Building on concrete experiences, students explain $\frac{1}{360}$ of a circle, called a "one-degree angle," is the unit for measuring angles (**MP.7**). Students connect their concrete measuring experiences with a new tool, the protractor and use it to more precisely determine angle measures (**MP.5**, **MP.6**). When solving word problems involving angle measures, students use drawings and tools to make sense of the problem, recognizing non-overlapping angles can be added or subtracted to find missing angles (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standards	Clarifications
KY.4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures. **MP.5**, **MP.6**	Coherence KY.3.G.1→KY.4.G.1
KY.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence of absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **MP.7**	Coherence KY.3.G.1→KY.4.G.2→KY.5.G.3
KY.4.G.3 Identify lines of symmetry. a. Recognize a line of symmetry for a two-dimensional figure. b. Identify line-symmetric figures and draw lines of symmetry. **MP. 5**, **MP.7**	

Attending to the Standards for Mathematical Practice

Using technology, using straightedges and/or protractors, students draw points, lines, line segments, rays, angles and perpendicular and parallel lines (**MP.5**). Students reason about the possible relationship of two lines or line segments. For example, students might use technology, uncooked spaghetti, or lines drawn on two transparency strips, to arrange two lines in different ways to determine possible events (the two lines might intersect, might intersect and be perpendicular, or may be parallel) (**MP.7**). Students analyze, compare and sort polygons based on their sides, angles and symmetry, explaining whether an attribute is a defining characteristic of that shape (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1
Common Addition and Subtraction Situations[1]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	**Group Size Unknown**	**Number of Groups Unknown**
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3
Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\, x\, (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$
If $a > b$ and $b > c$ then $a > c$
If $a > b$, then $b < a$
If $a > b$, then $–a < -b$
If $a > b$, then $a \pm c > b \pm c$
If $a > b$ and $c > 0$, then $a \times c > b \times c$
If $a > b$ and $c < 0$, then $a \times c < b \times c$
If $a > b$ and $c > 0$, then $a \div c > b \div c$
If $a > b$ and $c < 0$, then $a \div c < b \div c$

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★