INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states and other parts of the world

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
  - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;
  - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
  - Become self-sufficient individuals;

    - Become responsible members of a family, work group or community as well as an effective participant in community service;
    - Think and solve problems in school situations and in a variety of situations they will encounter in life;
    - Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
    - Express their creative talents and interests in visual arts, music, dance, and dramatic arts.
- Increase student attendance rates.
- Reduce dropout and retention rates.
- Reduce physical and mental health barriers to learning.
- Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> *(Model Curriculum Framework,* page 19*)*

<u>Legal Basis</u>

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

<u>704 KAR 8:040</u> Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;
- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and

- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.

- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

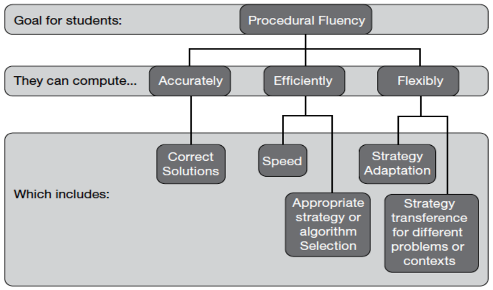
Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).
- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)
- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.



Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.

- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

The Modeling Process

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

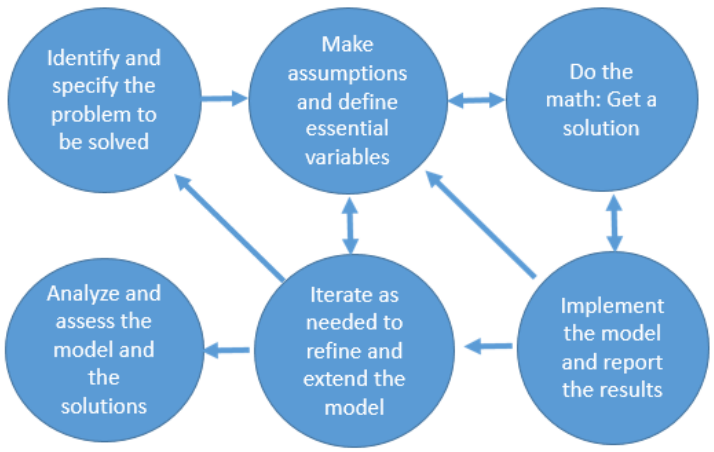
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

<u>The Kentucky Academic Standards (KAS) are Standards, not Curriculum</u>

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate "future-oriented" thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

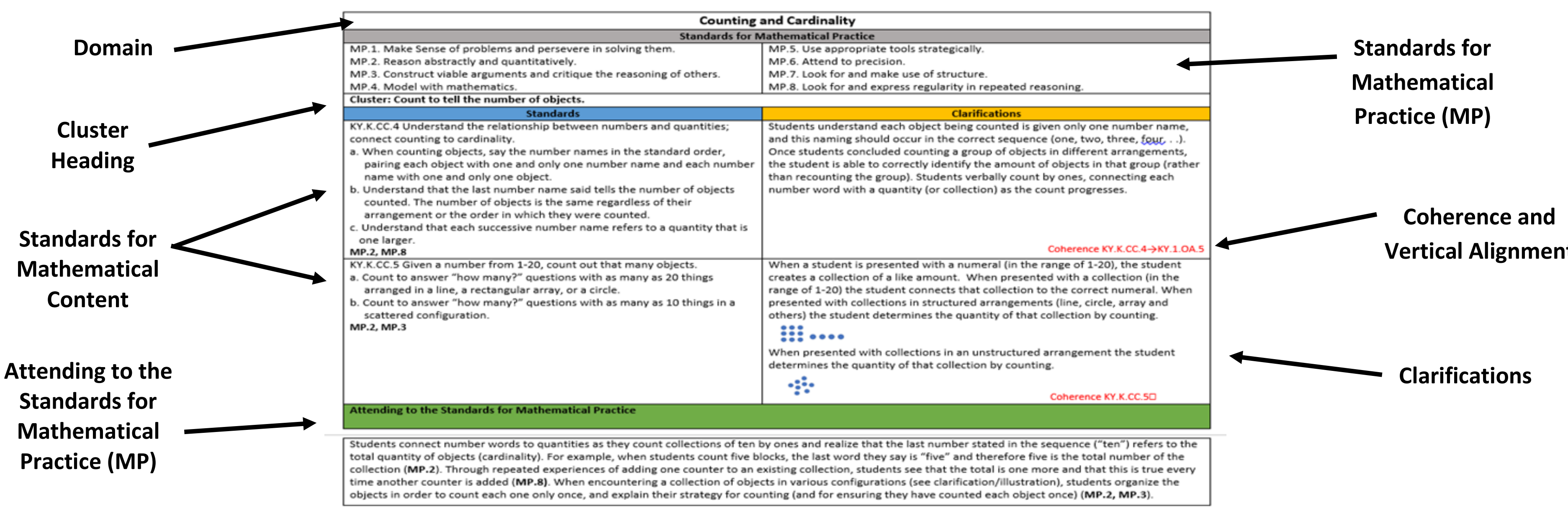
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

Standards for Mathematical Content define what students should understand and be able to do.

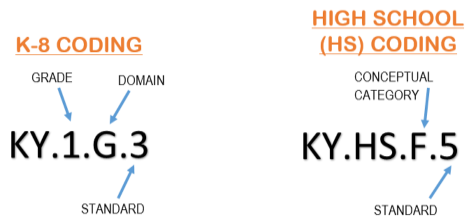
Standards for Mathematical Practice define how students engage in mathematical thinking.

The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.



Counting and Cardinality	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Count to tell the number of objects.	
Standards	**Clarifications**
KY.K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **MP.2, MP.8**	Students understand each object being counted is given only one number name, and this naming should occur in the correct sequence (one, two, three, four, . .). Once students concluded counting a group of objects in different arrangements, the student is able to correctly identify the amount of objects in that group (rather than recounting the group). Students verbally count by ones, connecting each number word with a quantity (or collection) as the count progresses. Coherence KY.K.CC.4→KY.1.OA.5
KY.K.CC.5 Given a number from 1-20, count out that many objects. a. Count to answer "how many?" questions with as many as 20 things arranged in a line, a rectangular array, or a circle. b. Count to answer "how many?" questions with as many as 10 things in a scattered configuration. **MP.2, MP.3**	When a student is presented with a numeral (in the range of 1-20), the student creates a collection of a like amount. When presented with a collection (in the range of 1-20) the student connects that collection to the correct numeral. When presented with collections in structured arrangements (line, circle, array and others) the student determines the quantity of that collection by counting. When presented with collections in an unstructured arrangement the student determines the quantity of that collection by counting. Coherence KY.K.CC.5□
Attending to the Standards for Mathematical Practice	
Students connect number words to quantities as they count collections of ten by ones and realize that the last number stated in the sequence ("ten") refers to the total quantity of objects (cardinality). For example, when students count five blocks, the last word they say is "five" and therefore five is the total number of the collection (**MP.2**). Through repeated experiences of adding one counter to an existing collection, students see that the total is one more and that this is true every time another counter is added (**MP.8**). When encountering a collection of objects in various configurations (see clarification/illustration), students organize the objects in order to count each one only once, and explain their strategy for counting (and for ensuring they have counted each object once) (**MP.2, MP.3**).	

How to Read the Coding of the Standards



Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council's 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Supplementary Materials to the Standards

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

Appendix A: Tables

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

Appendix B: Writing and Review Teams

Kentucky Academic Standards for Mathematics: Grade 5 Overview

Operations and Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Write and interpret numerical expressions. • Analyze patterns and relationships.	• Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths.	• Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	• Convert like measurement units within a given measurement system. • Understand and apply the statistics process. • Geometric measurement: understand concepts of volume and relate volume to multiplications and to addition.	• Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.

In grade 5, instructional time should focus on three critical areas:

1. **In the Numbers and Operations - Fractions and Operations and Algebraic Thinking domains, students will:**
    - apply their knowledge of fractions and fraction models to illustrate the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators;
    - establish fluency in calculating sums and differences with fractions and make a reasonable estimate of those sums and differences;
    - use the meaning of fractions, of multiplication and division, and the relationship between those operations to understand and explain why the procedures for multiplying and dividing fractions make sense.

    (Note: This is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. **In the Operations and Algebraic Thinking and Number and Operations in Base Ten, students will:**
    - develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations;
    - apply understandings of models for decimals, decimal notation and properties of operations to add and subtract decimals to hundredths;
    - develop fluency with decimal computations to hundredths and make reasonable estimates of their computation;
    - use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers to understand and explain why the procedures for multiplying and dividing finite decimals make sense.
3. **In the Measurement and Data and Geometry domains, students will:**
    - recognize volume as an attribute of three-dimensional space;
    - understand that a 1-unit by 1-unit cube is the standard unit for measuring volume;
    - understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps;
    - choose appropriate units, strategies and tools for solving problems which involve estimating and measuring volume;
    - decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes;
    - measure attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Write and interpret numerical expressions.	
Standards	**Clarifications**
KY.5.OA.1 Use parentheses, brackets or braces in numerical expressions and evaluate expressions that include symbols. **MP.1**, **MP.3**	Students work with the order of first evaluating terms in parentheses, then brackets, [] and then braces, {}. Coherence KY.5.OA.1→ KY.6.EE.2
KY.5.OA.2 Write simple expressions with numbers and interpret numerical expressions without evaluating them. **MP.2**, **MP.7**	Students translate from words “add 8 and 7, then multiply by 2” to 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. KY.6.EE.2 Coherence KY.4.OA.1→ KY.5.OA.2→KY.6.EE.3 KY.6.EE.4

Attending to the Standards for Mathematical Practice

Students move between words and symbols, understanding equivalent ways to express a statement. Students interpret the statement “The sum of 347, 124 and 99, divided by 30 as, (347 + 124 + 99) ÷ 30 and as $\frac{347+124+99}{30}$ (**MP.7**). As they evaluate such expressions, they realize there are options within the order of operations. In this expression, they add the three values and then divide by 30, or divide each addend by 30 and get the same answer. They think of a context to convince themselves two options will lead to the same answer (**MP.2**). In this case, students consider the two options and see the first idea is less ‘messy’ and therefore, a good choice (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Analyze patterns and relationships.	
Standards	**Clarifications**
KY.5.OA.3 Generate numerical patterns for situations. a. Generate a rule for growing patterns, identifying the relationship between corresponding terms (x, y). b. Generate patterns using one or two given rules (x, y). c. Use tables, ordered pairs and graphs to represent the relationship between the quantities. **MP.2**, **MP.4**	Given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, students generate terms in the resulting sequences (creating ordered pairs). Students observe the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. Graph the ordered pairs on a coordinate plane. Coherence KY.4.OA.5→ KY.5.OA.3→KY.6.EE.9.
Attending to the Standards for Mathematical Practice	
Students notice when they apply a rule, like add 3, several patterns emerge. The explicit pattern is the new value is 3 more than the original value. But, as they explore they notice if they pick an input that is 5 more than the last input, then the output is also 5 more. They reason about this contextually, for example thinking of people ages in three years. So, if they have a sibling that is 5 years older now, in three years, they will still be 5 years older (**MP.2**). They represent these patterns on graphs and use the graphs to make sense of the situation (**MP.4**)	

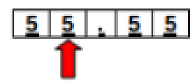
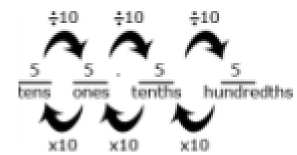
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten

Note: grade 5 expectations in this domain are limited to decimals through the thousandths place.

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand the place value system.

Standards	**Clarifications**
KY.5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. **MP.2**, **MP.7**	In the number 55.55, each digit is 5, but the value of each digit is different because of the placement.  The arrow points to is 1/10 of the 5 to the left and 10 times greater than the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times greater than five tenths.  Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.4.NBT.1→KY.5.NBT.1
KY.5.NBT.2 Multiply and divide by powers of 10. ● Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. ● Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. ● Use whole-number exponents to denote powers of 10. **MP.3**, **MP.8**	Students recognize when a number is multiplied by 10, a zero is added to the end because each digit's value became 10 times larger. Students use the same reasoning to explain in the problem. ● $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places. ● $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places. ● $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.5.NBT.2→ KY.6.EE.1

Standards	Clarifications
KY.5.NBT.3 Read, write and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **MP.2**, **MP.5**, **MP.7**	a. For the number 347.392… ● number name: three hundred forty-seven and three hundred ninety-two thousandths ● expanded form: $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$ Students relate numbers they are comparing back to common benchmarks of 0, $\frac{1}{2}$ (0.5, 0.50 and 0.500) and 1. When comparing numbers, 0.35 and 0.12, students make the connection 0.35 > 0.12, but also see the relationship of 0.12 < 0.35. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. KY.4.NBT.2 Coherence KY.4.NF.7→ KY.5.NBT.3
KY.5.NBT.4 Use place value understanding to round decimals to any place. **MP.5**, **MP.7**	Students go beyond application of an algorithm or procedure when rounding. Students demonstrate a deeper understanding of number sense and place value and explain and reason about the answers they get when they round. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.4.NBT.3→ KY.5.NBT.4

Attending to the Standards for Mathematical Practice
Students compare the value of the digits based on where they are in a number (**MP.7**). They reason 10 tens equal 100, 70 tens equal 700 and this can be illustrated with base 10 blocks or other visuals (**MP.2**). Students look across series of problems to notice a pattern when multiplying by 10, 100 or 1000 (**MP.8**) and justify why patterns exist (why 36 x 100 = 3600), rather than superficially noting 'you add zeros,' they explain or show there are actually 36 *hundreds*, so 3600 (**MP.3**). Students use similar reasoning to compare decimal values, explaining tenths are larger than hundredths and therefore, they look to first see which values have more tenths before looking at how many hundredths it has (**MP.2**, **MP.7**). Students use tools such as number lines and base 10 blocks to see place value relationships with decimals in order to compare and to round (**MP.5**).

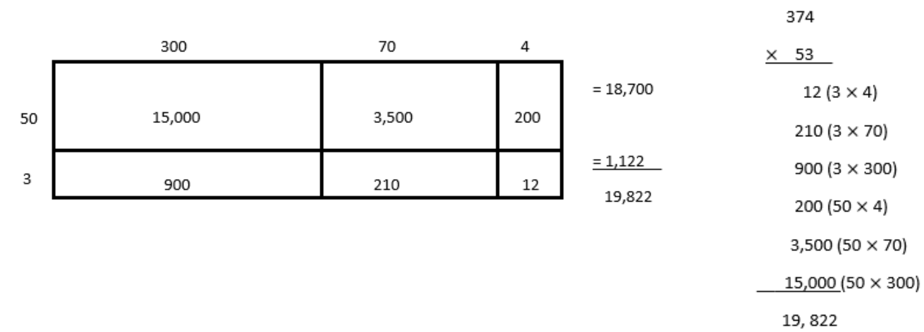
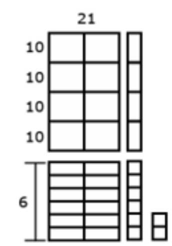
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

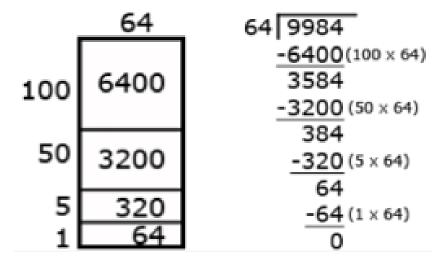
Number and Operations in Base Ten

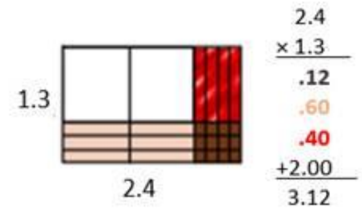
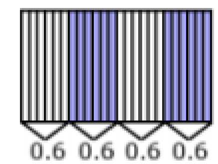
Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standards	Clarifications
KY.5.NBT.5 Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm. **MP.7**, **MP.8**	Students make connections from previous work with multiplication, using models/representations to develop an efficient algorithm to multiply multi-digit whole numbers.  Coherence KY.4.NBT.5→ KY.5.NBT.5→KY.6.NS.3
KY.5.NBT.6 Divide up to four-digit dividends by two-digit divisors. a. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using… ● strategies based on place value ● the properties of operations ● the relationship between multiplication and division b. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. **MP.2**, **MP.3**, **MP.4**	Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of partitive and measurement models for division to an algorithm, including partial quotients. Some examples include: 968 ÷ 21 = Students use base ten models by representing 962 and use the model to make an array with one dimension of 21. Student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array. 

Standards	Clarifications
	Students use an area model for division shown below. As the student uses the area model, s/he keeps track of how much of the 9,984 is left to divide.  Students use expanded notation 2682 ÷ 25 = (2000 + 600 +80 + 2) ÷ 25. Students use his or her understanding of the relationship between 100 and 25, to think “I know 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80. Then 600 divided by 25 has to be 24. Since 3 × 25 is 75, I know that 80 divided by 25 is 3 with a remainder of 5. I can’t divide 2 by 25 so 2 plus the 5 leaves a remainder of 7. 80 + 24 + 3 =107. So the answer is 107 with a remainder of 7.” Students use an equation that relates division to multiplication, 25 × n = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 × 100 =2500. Coherence KY.4.NBT.6→ KY.5.NBT.6→KY.6.NS.2
KY.5.NBT.7 Operations with decimals to hundredths. a. Add, subtract, multiply and divide decimals to hundredths using… ● concrete models or drawings ● strategies based on place value ● properties of operations ● the relationship between addition and subtraction b. Relate the strategy to a written method and explain the reasoning used. **MP.2**, **MP.3**, **MP.5**	Students connect previous experiences with the meaning of multiplication and division of whole numbers to multiplication and division of decimals using estimation, models and place value structure. For example: 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.  The answer is 3 and $\frac{7}{10}$ or 3.7 An area model can be used for illustrating products.

Standards	Clarifications
	 Students describe the partial products displayed by the area model. For example, "$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$. $\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$. 1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$. 1 group of 2 is 2." Students dividing decimals for example could find the number in each group or share by applying the fair sharing model or separating decimals in to equal parts such as 2.4 ÷ 4 = 0.6  Coherence KY.4.NBT.6→ KY.5.NBT.7→KY.6.NS.3

Attending to the Standards for Mathematical Practice

Students understand when given a multiplication problem, they have a choice in how they solve it and select a way that makes sense for the values in the problem. For example, for 1234 x 12, they see the small numbers lend to a break apart strategy and solve the problem this way:

1234 x 10 = 12340

1234 x 2 = 2468

Then add the partial products to equal 14,808 (**MP.7**). Other students may stack the two values and use an algorithm. Students recognize a rectangle is an effective model for ensuring all partial products are calculated, for both whole numbers and decimals (**MP.4**). As students explore problems with decimal values, they reason about the problem, rather than following rules devoid of meaning (count the number of decimal places). For example, when multiplying 4 x 1.5, they use a break apart strategy, as they have for whole numbers, noticing 4 x 1 = 4 and 4 x 0.5 = 2, so therefore, 4 x 1.5 = 6 (**MP.2**). They explain why this works and when they use this strategy (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations - Fractions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use equivalent fractions as a strategy to add and subtract fractions.

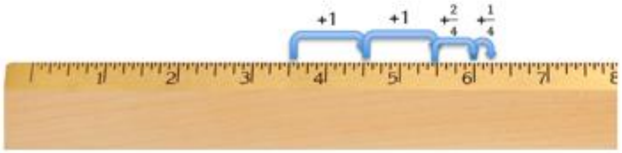
Standards	Clarifications
KY.5.NF.1 Efficiently add and subtract fractions with unlike denominators (including mixed numbers) by… ● using reasoning strategies, such as counting up on a number line or creating visual fraction models ● finding common denominators **MP.2**, **MP.3**	Using common denominator $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad+bc)}{bd}$ KY.4.NF.1 Coherence KY.4.NF.3→ KY.5.NF.1→KY.6.EE.7
KY.5.NF.2 Solve word problems involving addition and subtraction of fractions. a. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. b. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. **MP.1**, **MP.4**	a. For example: Mary ate $\frac{1}{3}$ of the pizza. Tommy ate $\frac{2}{5}$ of the pizza. How much of the total pizza did they eat together? ● making equivalent fractions to add/subtract fractions ● using visual representations to add/subtract fractions ○ Area Model ○ Linear Model b. Recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$. Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.4.NF.3→KY.5.NF.2

Attending to the Standards for Mathematical Practice

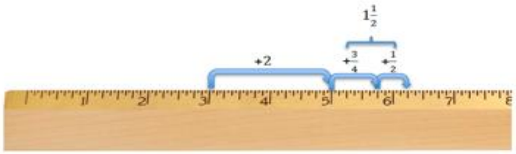
As students add and subtract fractions, they make sense of situations in story problems, selecting and creating representations of the situation such as partitioned rectangles or number lines (**MP.1**, **4**). Students notice if the fractions in the problem can be solved using a reasoning strategy, or if it is more efficient to find common denominators (**MP.2**). For example, for the problem $2\frac{3}{4} + 3\frac{1}{2}$, students may mentally or physically refer to a ruler and use a counting up strategy:

Attending to the Standards for Mathematical Practice

$2\frac{3}{4} + 3\frac{1}{2} + = 3\frac{1}{2} + 2 + \frac{3}{4}$



$2\frac{3}{4} + 3\frac{1}{2} = 3 + 2 + \frac{3}{4} + \frac{1}{2} = 5 + 1\frac{1}{4} = 6\frac{1}{4}$



Or, students use a break apart strategy noticing $\frac{3}{4}$ is $\frac{1}{2} + \frac{1}{4}$ and therefore, reason there are 6 wholes and $\frac{1}{4}$ more, so $6\frac{1}{4}$ is the sum. Other students rewrite the fractions as $2\frac{3}{4} + 3\frac{2}{4}$ and add the whole numbers and fractions separately and then combine them. Students explain their reasoning strategies and students listen to others who solved the problem differently than they solved it and determine if the reasoning makes sense, if it is efficient and if the answer is correct (**MP.3**).

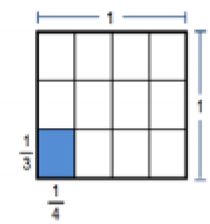
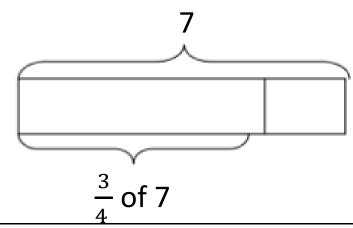
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

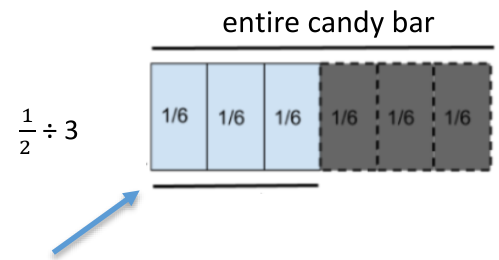
Number Operations - Fractions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standards	Clarifications
KY.5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. **MP.4**, **MP.8**	For example students interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3 and when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. Coherence KY.5.NF.3→KY.6.RP.2
KY.5.NF.4 Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(\frac{a}{b})$ x q as parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x $q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. **MP.1**	a. Students use a visual fraction model to show $(\frac{2}{3})$ x 4 = $\frac{8}{3}$ and create a story context for this equation. Do the same with $(\frac{2}{3})$ x $(\frac{4}{5})$ = $\frac{8}{15}$. (In general, $(\frac{a}{b})$ x $(\frac{c}{d})$ = $\frac{ac}{bd}$.)  b. For example the shaded portion shows the rectangle with the appropriate unit fraction side lengths. Coherence KY.4.NF.4→ KY.5.NF.4→KY.6.G.1
KY.5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a	$\frac{1}{4}$ x 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7. 

Standards	Clarifications
fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1. **MP.2**, **MP.6**	Coherence KY.4.OA.1→ KY.5.NF.5→KY.6.RP.1
KY.5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers. **MP.4**, **MP.5**	KY.5.MD.2 Coherence KY.4.NF.4→KY.5.NF.6
KY.5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. b. Interpret division of a whole number by a unit fraction and compute such quotients. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. **MP.1**, **MP.4**, **MP.8**	Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of division of whole numbers to divide whole numbers by unit fractions and unit fractions by whole numbers. Division of a fraction by a fraction is not a requirement at grade 5. a. Create a story context for $(\frac{1}{3}) \div 4$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$. b. Create a story context for $4 \div (\frac{1}{5})$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$, because $20 \times (\frac{1}{5}) = 4$. c. By using visual fraction models and equations to represent the problem.  Each child will get one piece. Half to be shared with 3 students. Coherence KY.4.NF.4→ KY.5.NF.7→KY.6.NS.1

Attending to the Standards for Mathematical Practice
Students look for repeated reasoning in order to understand the meaning of the operations (**MP.8**). Rather than memorize rules that do not make sense, students use mathematical representations to consider the relative size of their answers (**MP.4**). For example, students solve the classic "brownie sharing" problems, wherein brownies are shared equally with children. In considering how 4 children share 5 brownies. They use drawings of rectangles and partition to show each child will get $1\frac{1}{4}$ brownies. As students continue to explore brownie sharing, they notice patterns. In this case, they see $5 \div 4$ means the same as $\frac{5}{4}$ (**MP.4**). Students reason quantitatively as they work on scaling problems in context (**MP.2**). For example, in $\frac{3}{4}$ of 16, students might reason the answer is less than 16. To solve it, they begin by thinking $\frac{1}{4}$ of 16 is 4, then think 3 groups of 4 is 12. As students divide a problem such as $4 \div \frac{1}{8}$, $7 \div \frac{1}{8}$, $10 \div \frac{1}{8}$, they notice how many eighths in one whole and then multiply by how many wholes they have. This pattern leads to an understanding of why it looks like they are multiplying by the denominator (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Convert like measurement units within a given measurement system.	
Standards	**Clarifications**
KY.5.MD.1 Convert among different size measurement units (mass, weight, liquid volume, length, time) within one system of units (metric system, U.S. standard system and time). **MP.3**, **MP.8**	Within the same system convert measurements in a larger unit in terms of a smaller unit and a smaller unit in terms of a larger unit. Use these conversions in solving multi-step, real world problems. Coherence KY.4.MD.1→ KY.5.MD.1→KY.6.RP.3
Attending to the Standards for Mathematical Practice	
Students notice patterns about how units and measurements relate to each other (**MP.8**). For example, students measure various objects in meters and in centimeters (using a meter stick). As they measure their items, they record the measurements in a table. They notice the object that measures about 300 centimeters also measures about 3 meters (**MP.8**). They explain why this pattern is true, arguing each of the meters has 100 centimeters, so 3 meters will have 300 centimeters and more generally explaining the smaller the unit the more of unit there will be when measuring the same object (**MP.3**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply the statistics process.	
Standards	**Clarifications**
KY.5.MD.2 Identify and gather data for statistical questions focused on both categorical and numerical data. Select an appropriate data display (bar graph, pictograph, dot plot). Make observations from the graph about the questions posed. **MP.4**, **MP.5**, **MP.6**	Generate questions for which data can be gathered and sort questions that are categorical (Possible question: What is your favorite after-school activity?) and questions that are numerical (Possible question: How many times can you say/write your name in one minute?). After gathering data on a question, students discuss which graphs are possible and which ones are not possible, and why. Students select one type of graph that fits the data gathered and create the graph, by hand or by using technology. KY.6.SP.2 Coherence KY.4.MD.4→ KY.5.MD.2→KY.6.SP.4
Attending to the Standards for Mathematical Practice	
After gathering data on a question of interest, students recognize they have many data points and therefore, decide they will do a scaled graph (**MP.4**). In creating the graph, they decide to do a picture graph and pick a scale of 1 picture = 4 data points (**MP.6**). In another situation, students recognize they have numerical data and create a dot plot and decide to use a spreadsheet on the computer to create the graph (**MP.5**). Students compare how dot plots and bar graphs are similar and different, recognizing when to use each (**MP.6**).	

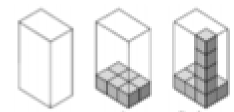
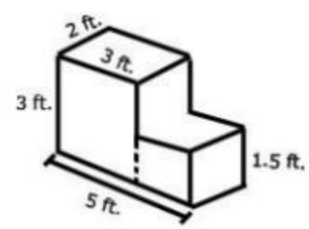
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Standards	Clarifications
KY.5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **MP.6**	a.   b. Coherence KY.3.MD.5→ KY.5.MD.3
KY.5.MD.4 Measure volumes by counting unit cubic cm, cubic in, cubic ft. and improvised units. **MP.5**, **MP.6**	Coherence KY.3.MD.6→ KY.5.MD.4
KY.5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes. b. Apply the formulas *V= l x w x h* and *V = B x h* for rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by	For example, students determine the volume of concrete needed to build the steps in the diagram below.  Coherence KY.4.MD.3→ KY.5.MD.5→KY.6.G.2

Standards	Clarifications
adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **MP.1**, **MP.4**, **MP.8**	

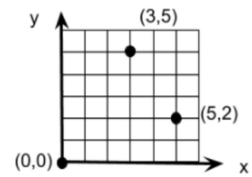
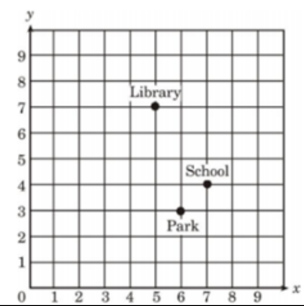
Attending to the Standards for Mathematical Practice
Students use cubes to cover a bottom layer of a rectangular prism, understanding cube as a unit cube (**MP.5**). As students place the cubes in layers to fill the rectangular solid, they notice the number of cubes in each layer can be found by multiplying [number of cubes in one row] x [number of rows] and this product (the base) can be multiplied by how many layers to determine how many unit cubes will fill the container (**MP.8**). Students connect this idea to the formulas for volume and use these formulas to solve problems (**MP.4**). When a three-dimensional shape is not a single rectangular solid, students analyze the shape and its measurements to determine how to decompose the shape and find the volume of each prism (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Graph points on the coordinate plane to solve real-world and mathematical problems.

Standards	Clarifications
KY.5.G.1 Use a pair perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second. **MP.4**, **MP.7**	This standard pertains to the first quadrant only which limits to positive ordered pairs only.  Coherence KY.5.G.1→KY.6.NS.6
KY.5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation. **MP.1**, **MP.6**	For example, students use the coordinate grid, which ordered pair represents locations of places or objects.  KY.6.NS.8 Coherence KY.5.G.2→KY.6.G.3

Attending to the Standards for Mathematical Practice

Students notice a coordinate axis, is in fact, coordinating a horizontal number line with a vertical number line (**MP.7**). These two lines need a title, scale and a label in order to be understood by a reader (**MP.6**). Students record data in their graph from exploring a pattern and gain insights about the pattern. For example, students graph data from a two-column table that shows the cost of buying pineapples (one pineapple costs $2, three pineapples costs $6) and use the coordinate axis to explain what they notice about the relationship between the number of pineapples and the cost of pineapples (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Classify two-dimensional figures into categories based on their properties.

Standards	Clarifications
KY.5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. **MP.3**, **MP.6**	For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. Coherence KY.4.G.2→ KY.5.G.3
KY.5.G.4 Classify two-dimensional figures in a hierarchy based on properties. **MP.1**, **MP.7**	Figures from previous grades: polygons, rhombus/rhombi, rectangle, square, triangle quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter, circle. For example: • Polygon - a closed plan figure formed from line segments that meet only at their endpoints. • Quadrilateral - a four-sided polygon • Rectangle - a quadrilateral with two pairs of congruent parallel sides and four right angles. • Rhombus - a parallelogram with all four sides equal in length • Square - a parallelogram with four congruent sides and four right angles. Coherence KY.4.G.2→ KY.5.G.4

Attending to the Standards for Mathematical Practice

As they have done in grade 3, students describe attributes they notice for a particular type of quadrilateral, focusing on side lengths and angles (**MP.6**). They compare the lists of defining attributes across shapes to notice what they have in common and what is different**.** (**MP.7**). They explain some types of quadrilaterals (parallelograms) are also rectangles because all the attributes of a parallelogram are also attributes of a rectangle (**MP.3**). They use this analysis to build an understanding of a rectangle as a special case of a parallelogram (a parallelogram with 90 degree angles) and use these understandings to create a hierarchy of quadrilaterals (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1
Common Addition and Subtraction Situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	Group Size Unknown	Number of Groups Unknown
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3
Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\, x\, (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c then a > c
If a > b, then b < a
If a > b, then –a < -b
If a > b, then a ± c > b ± c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a ÷ c > b ÷ c
If a > b and c < 0, then a ÷ c < b ÷ c

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★