INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states and other parts of the world

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
  - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;
  - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
  - Become self-sufficient individuals;

    - Become responsible members of a family, work group or community as well as an effective participant in community service;
    - Think and solve problems in school situations and in a variety of situations they will encounter in life;
    - Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
    - Express their creative talents and interests in visual arts, music, dance, and dramatic arts.
- Increase student attendance rates.
- Reduce dropout and retention rates.
- Reduce physical and mental health barriers to learning.
- Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> (*Model Curriculum Framework,* page 19)

Legal Basis

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

704 KAR 8:040 Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;
- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and

- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.

- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

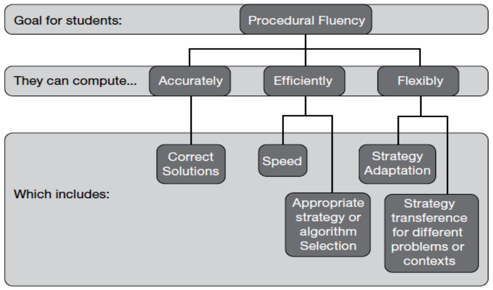
Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).
- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)
- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.



Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.

- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

The Modeling Process

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

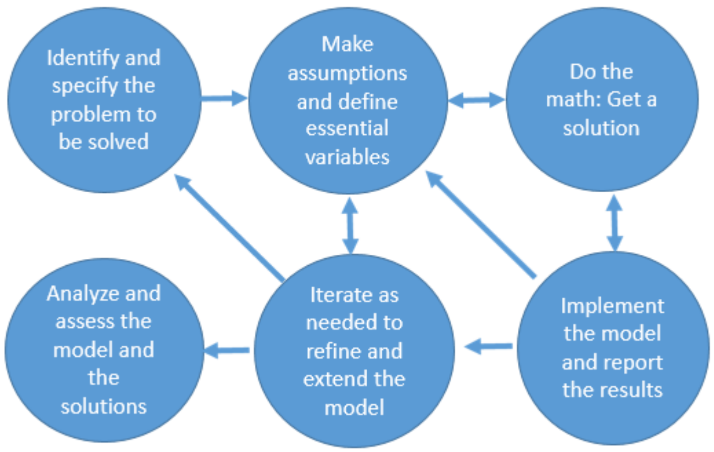
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

<u>The Kentucky Academic Standards (KAS) are Standards, not Curriculum</u>

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate "future-oriented" thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

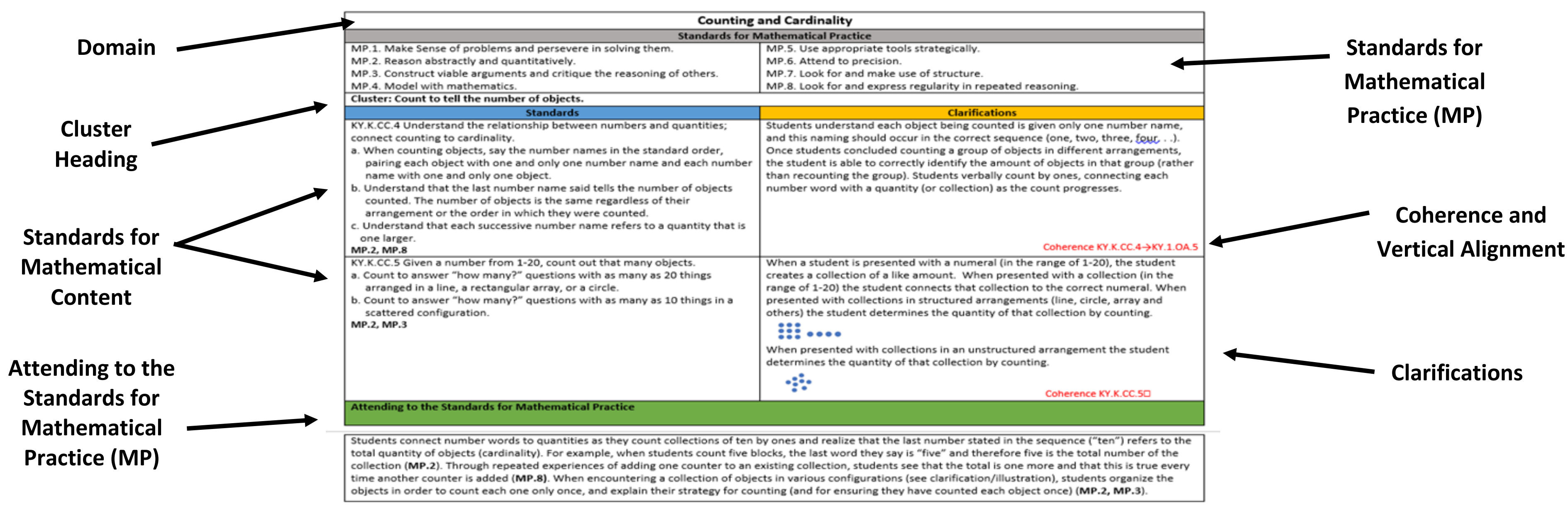
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

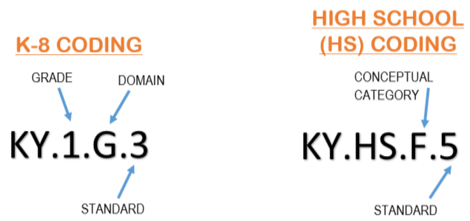
Standards for Mathematical Content define what students should understand and be able to do.

Standards for Mathematical Practice define how students engage in mathematical thinking.

> *The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.*



How to Read the Coding of the Standards



Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council's 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Supplementary Materials to the Standards

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

Appendix A: Tables

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

Appendix B: Writing and Review Teams

Kentucky Academic Standards for Mathematics: Grade 6 Overview

Ratios and Proportional Relationships (RP)	The Number System (NS)	Expressions and Equations (EE)	Geometry (G)	Statistics and Probability (SP)
• Understand ratio concepts and use ratio reasoning.	• Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Multiply and divide multi-digit numbers and find common factors and multiples. • Apply and extend previous understanding of numbers to the system of rational numbers.	• Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.	• Solve real-world and mathematical problems involving area, surface area and volume.	• Develop understanding of the process of statistical reasoning. • Develop understanding of statistical variability. • Summarize and describe distributions.

In grade 6, instructional time should focus on four critical areas:

1. In the Ratios and Proportional Relationships domain, students will:

- use reasoning about multiplication and division to solve ratio and rate problems about quantities;
- connect understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from and extending, pairs of rows (or columns) in the multiplication table and by analyzing simple drawings that indicate the relative size of quantities; and
- expand the scope of problems for which they can use multiplication and division to solve problems and they connect ratios and rates.

2. In the Number System domain, students will:

- use the meaning of fractions and relationships between multiplication and division to understand and explain why the procedures for dividing fractions make sense;
- extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, particularly negative integers; and
- reason about the order and absolute value of rational numbers and about the location of points on a coordinate plane.

3. In the Expressions, Equations and Inequalities domain, students will:

- write expressions and equations that correspond to give situations, using variables to represent an unknown and describe relationships between quantities;
- understand that expressions in different forms can be equivalent and use the properties of operations to rewrite and evaluate expressions in equivalent forms; and
- use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations.

4. In the Geometry domain, students will:

- reason about relationships among shapes to determine area, surface area and volume. They find areas of right triangles, other triangles and special quadrilaterals by decomposing these shapes, rearranging or removing pieces and relating the shapes to rectangles.
- discuss, develop and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths

5. In the Statistics and Probability domain, students will:

- develop their ability to think statistically;
- recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally and also in the sense that it is a balance point.
- recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.
- learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps and symmetry, considering the context in which the data were collected.

Ratios and Proportional Relationships	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understanding ratio concepts and use ratio reasoning to solve problems.	
Standards	**Clarifications**
KY.6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **MP.2**, **MP.6**	Students use the concept of ratios as a comparison between related quantities; students also express these relationships in equivalent ratios in lowest terms, where appropriate. Coherence KY.5.NF.5→KY.6.RP.1
KY.6.RP.2 Understand the concept of a unit rate a/b associated with a ratio *a:b* with B ≠ 0 and use rate language in the context of a ratio relationship. **MP.2**, **MP.6**	Expectations for unit rates in grade 6 are limited to non-complex fractions; additionally, students reduce ratios of two whole numbers to a unit rate involving a fraction and a denominator of 1. Students describe real-life contexts using ratio language. Coherence KY.5.NF.3→KY.6.RP.2→KY.7.RP.1
KY.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve rate problems including those involving unit pricing and constant speed. c. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **MP.1**, **MP.4**, **MP.7**	a. Students find the missing values in a table, assuming the values in the table represent a proportional relationship; students plot the values from a table on a coordinate plane, with appropriate labels and scales; Students compare the ratios of tables, answering, which has a greater/less rate. b. Students find a unit rate from a given situation and reason to apply it to a future scenario. c. For example, convert miles per hour to feet per hour or meters per minute to meters per hour using appropriate conversion ratios. Coherence KY.6.RP.3→KY.7.RP.2

Attending to the Standards for Mathematical Practice
As students solve similar problems, they develop their skills in several mathematical practice standards, reasoning abstractly and quantitatively (**MP.2**), abstracting information from the problem, creating a mathematical representation of the problem and correctly working with both part-part and part-whole situations. Students attend to precision (**MP.6**) as they properly use ratio notation, symbolism and label quantities. Representing ratios in various ways help students see the additive and multiplicative structure of ratios (**MP.7**). Students model with mathematics (**MP.4**) when they solve real-world and mathematical problems using ratio and rate reasoning, especially when they make use of various representations in the modeling process.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
Standards	**Clarifications**
KY.6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. **MP.1**, **MP.2**, **MP.3**	For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient: How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mile? Coherence KY.5.NF.7→KY.6.NS.1→KY.7.NS.2
Attending to the Standards for Mathematical Practice	
Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" (**MP.1**). For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. Students confirm, for example, that 10 ÷ 2 can be found by determining how many groups of two are in ten. They apply that strategy to the division of fractions (**MP.3**). Students use pictorial representations such as area models, array models, number lines and drawings to conceptualize and solve problems.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.	
Standards	**Clarifications**
KY.6.NS.2 Fluently divide multi-digit numbers using an algorithm. a. Convert a rational number to a decimal using long division. b. Know that the decimal form of a rational number terminates in 0s or eventually repeats. **MP.7**, **MP.8**	a. Divide a rational number a/b using long division, making sure to include rational numbers equivalent to terminating decimals and rational numbers equivalent to repeating decimals. b. Students understand and explain when they have a 0 remainder in a long division problem, the quotient (answer) is a terminating decimal; students understand when they notice a pattern in the process of dividing, they conclude they will never reach a 0 remainder and they then notate the part of the quotient that is repeating by marking a bar over those values. Coherence KY.5.NBT.6→KY.6.NS.2
KY.6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. **MP.2**, **MP.6**	Emphasis is on the role of the decimal point in operations and how place value is critical to the overall fluency of the performed operations involving decimals. KY.5.NBT.5 Coherence KY.5.NBT.7→KY.6.NS.3→KY.7.NS.3
KY.6.NS.4 Use the distributive property to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **MP.8**	Express numerical expressions using the distributive property; understand there may be multiple equivalent expressions, but only one will have been completely factored (the greatest common factor removed using the distributive property) such as 6 + 21 = 3 (2 +7). Coherence KY.4.OA.4→KY.6.NS.4

Attending to the Standards for Mathematical Practice
Students understand and use connections between divisibility and the greatest common factor to apply the distributive property (**MP.2**). Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students use precise language and place value (**MP.6**) when adding, subtracting, multiplying and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, 326.31 is read as three hundred twenty-six and thirty-one hundredths (**MP.7**). Students calculate sums, differences, products and quotients of decimal numbers with a degree of precision appropriate to the problem context.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understanding of numbers to the system of rational numbers.	
Standards	**Clarifications**
KY.6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **MP.1. MP.2**, **MP.4**	For example, positive and negative temperatures or elevations, with the understanding that zero means the freezing point Celsius of water or sea level. Coherence KY.6.NS.5→KY.7.NS.1
KY.6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes, using appropriate range and intervals, to represent points on the line and in the plane, that include negative numbers and coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize 0 is its own opposite and the opposite of a negative number is a positive, and the opposite of a negative number is a positive, such as $-(-3) = 3$. b. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. c. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize the similarity between whole numbers, their negative opposites and their positions on a number line, ordered pairs differ only by signs and their locations on one or both axes. **MP.2**, **MP.4**	a. Emphasis is on student understanding that every positive location on a number line has an opposite the same distance from zero in the negative direction and vice versa. Logically following from this is the fact that zero, as it has no positive or negative sign, is its own opposite. b. Emphasis is on generalizing patterns about where coordinates are located on a coordinate plane. c. The intent is for students to see a coordinate axis is the combination of a vertical number line and a horizontal number line. KY.6.EE.6 Coherence KY.5.G.1→KY.6.NS.6→KY.7.NS.1

KY.6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order. **MP.1. MP.2**, **MP.4**	a. Interpret two numbers, including two negatives, as one is to the left or right (or above or below) the other on a number line diagram. b. Understand, as with 6.NS.7a, positive and negative rational numbers represent real-life situations and can be compared. c. Interpret a positive or negative direction from zero as an absolute value, or magnitude, to describe a real-life situation. d. Recognize a number's distance from zero can be compared to another number's distance from zero with a "less than" or "greater than" distinction. Coherence KY.5.NBT.3→KY.6.NS.7→KY.7.NS.1 KY.6.EE.8
KY.6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **MP.5**, **MP.7**	For example, represent the vertices of a rectangle in the coordinate plane and find distances between horizontal and vertical vertices accurately. Given a vertex of (-2, 3), a length of 5 and a width of 11, locate the other three vertices of the rectangle. Coherence →KY.5.G.2→KY.6.NS.8

Attending to the Standards for Mathematical Practice
Students use vertical and horizontal number lines to visualize integers and better understand their connection to whole numbers. They divide number line intervals into sub-intervals of tenths to determine the correct placement of rational numbers (**MP.7**). Students may represent a decimal as a fraction or a fraction as a decimal to better understand its relationship to other rational numbers to which it is being compared (**MP.2**). To explain the meaning of a quantity in a real-life situation (involving elevation, temperature, or direction), students draw a diagram and/or number line to illustrate the location of the quantity in relation to zero or an established level that represents zero in that situation (**MP.4**). Students understand the placement of negative numbers on a number line by observing the patterns that exist between negative and positive numbers with respect to zero (**MP.7**). They recognize two numbers are opposites if they are the same distance from zero and zero is its own opposite. Students extend their understanding of the number line structure to the coordinate plane to determine a point's location. They recognize the relationship between the signs of a point's coordinates and the quadrant in which the point lies.

Expression and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.	
Standards	**Clarifications**
KY.6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. **MP.2**, **MP.6**	Interpret an exponent of size *n* as a repetitive multiplication expression of the base multiplied by itself *n* times; use the standard order of operations using exponents to evaluate numerical expressions. Coherence KY.5.NBT.2→KY.6.EE.1→KY.8.EE.1
KY.6.EE.2 Write, read and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sums, term, product, factor, quotient, coefficient); view one or more parts of an expression in a single entity. c. Evaluate expressions for specific values of their variables, including values that are non-negative rational numbers. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **MP.1**, **MP.3**, **MP.4**	For example, a. Express the calculation "y less than 5" as 5 – y. b. Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Use the formulas $V = s^3$ and $SA = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$ meter. KY.5.OA.1 Coherence KY.5.OA.2→KY.6.EE.2
KY.6.EE.3 Apply the properties of operations to generate equivalent expressions. **MP.7**, **MP.8**	Using Associative, Commutative and Distributive properties to generate equivalent expressions. Coherence KY.5.OA.2→KY.6.EE.3→KY.7.EE.1

KY.6.EE.4 Identify when two expressions are equivalent when the two expressions name the same number regardless of which value is substituted into them. **MP.2**, **MP.3**, **MP.7**	Students commonly think of variables as a missing number. The focus of this standard is recognizing the variable represents *any* number. In other words, they do not seek to find a single number to replace the letter, but they substitute any number and the expressions will be equivalent. When each expression (not just the variable) is altered by the same value, the expressions remain equivalent, no matter the value. Coherence KY.5.OA.2→KY.6.EE.4→KY.7.EE.1

Attending to the Standards for Mathematical Practice
Students connect symbols to their numerical referents. They understand exponential notation as repeated multiplication of the base number. Students realize 3^2 is represented as 3×3, with a product of 9 and explain how 3^2 differs from 3×2, where the product is 6. Students determine the meaning of a variable within a real-life context (**MP.2**). Students look for structure in expressions by deconstructing them into a sequence of operations. They make use of structure to interpret an expression's meaning in terms of the quantities represented by the variables. In addition, students make use of structure by creating equivalent expressions using properties. For example, students write $6x$ as $x + x + x + x + x + x$, $4x + 2x$, $3(2x)$, or other equivalent expressions (**MP.7**). Students look for regularity in a repeated calculation and express it with a general formula (**MP.8**). Students work with variable expressions while focusing more on the patterns that develop than the actual numbers that the variable represents. For example, students move from an expression such as $3 + 3 + 3 + 3 = 4 \cdot 3$ to the general form $m + m + m + m = 4 \cdot m$, or $4m$. Similarly, students move from expressions such as $5 \cdot 5 \cdot 5 \cdot 5 = 5^4$ to the general form $m \cdot m \cdot m \cdot m = m^4$. These are especially important when moving from the general form back to a specific value for the variable (**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason about and solve one-variable equation and inequalities.	
Standards	**Clarifications**
KY.6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **MP.1**, **MP.2**, **MP.7**	From a set of numbers, substitute values to choose which satisfy a given equation or inequality. An equation or inequality with no solutions from the list may be described as having no solutions or an empty set of solutions, given the set of possible values. Coherence KY.6.EE.5→KY.8.EE.8
KY.6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set. **MP.2**, **MP.6**	Represent an unknown quantity in real-world context appropriately with a variable and write an expression to show this. Coherence KY.6.EE.6→KY.7.EE.4
KY.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **MP.1**, **MP.2**, **MP.3**, **MP.4**	Emphasis is on understanding equations can be solved by using subtraction as an opposite operation of addition and division as an opposite operation of multiplication. Additionally, emphasis is on the importance of keeping the equations balanced when solving. Coherence KY.6.EE.7→KY.7.EE.4
KY.6.EE.8 Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of these forms have infinitely many solutions; represent solutions of such inequalities on vertical and horizontal number lines. **MP.3**, **MP.7**	Emphasis is on students understanding the phrases “more than”, “less than”, “at least” and “at most” represent constraints and conditions and are therefore associated with the operators listed in real-world problems. Students also understand an inequality does not yield a specific value, but rather an infinite range of values. Students also appropriately represent solutions to inequalities using both open and closed circles, along with direction, on vertical and horizontal number lines. Coherence KY.6.EE.8→KY.7.EE.4

Attending to the Standards for Mathematical Practice
Students have previously explored the concept of equality. In grade 6, students explore equations as one expression being set equal to a specific value. A solution is a value of the variable that makes the equation true and students may use various processes to identify such values that, when substituted for the variable, will make the equation true (**MP.2**). This reasoning is also applied when recognizing solutions for inequalities, such that students realize the value of a variable is one that would make the inequality statement true. Students use manipulatives and pictures (e.g., tape-like diagrams) to represent the equations and their solution strategies. When writing equations, students learn to be precise in their definition of a variable (**MP.6**), for example, writing “n equals John’s age in years” as opposed to writing “n is John”. Students use tables and graphs to compare different expressions or equations to make decisions in real-world scenarios. These models also create structure as students gain knowledge in writing expressions and equations (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and analyze quantitative relationships between dependent and independent variables.	
Standards	**Clarifications/Illustrations**
KY.6.EE.9 Use variables to represent two quantities in a real-world problem that changes in relationship to one another; a. Appropriately recognize one quantity as the dependent variable and the other as the independent variable. b. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. c. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the question. **MP.3**, **MP.4**, **MP.7**	Students understand in real-world problems, one quantity dependently changes relative to another independent quantity at a constant rate; understand, at times, the quantities given may not have a clear independent/dependent relationship. Coherence KY.5.OA.3→KY.6.EE.9→KY.8.EE.5
Attending to the Standards for Mathematical Practice	
Students show relationships between quantities with multiple representations, using language, a table, an equation, or a graph. Translating between multiple representations helps students understand each form represents the same relationship and provides a different perspective on the relationship. (**MP.3**) Students construct arguments supporting mathematical claims about the relationship between the dependent and independent variable using evidence from the different representations. Students are also equipped to examine the evidence and claims of other students while comparing the different representations. Students model with mathematics (**MP.4**) the relationship between dependent and independent variables. Students use many forms to represent the relationship between quantities. Students demonstrate a mathematical model by translating between multiple representations to provide different perspectives on the relationship at hand.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve real-world and mathematical problems involving area, surface area and volume.	
Standards	**Clarifications**
KY.6.G.1 Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and quadrilaterals; apply these techniques in the context of solving real-world and mathematical problems. **MP.1**, **MP.6**, **MP.8**	Area of the listed shapes may be thought of as a rectangle with larger area, subtracting the areas exterior to the actual shape to obtain the true area, or as a composite area of smaller triangles and rectangles which sum to the true area of the given shape. Students recognize given shapes can be combined to find area or decomposed to find area and one method may be more efficient than the other. Coherence KY.5.NF.4→KY.6.G.1→KY.7.G.6
KY.6.G.2 Find the volume of a right rectangular prism with rational number edge lengths. Apply the formulas *V = lwh* and *V = Bh* to find volumes of right rectangular prisms with rational number edge lengths in the context of solving real-world and mathematical problems. **MP.2**, **MP.5**, **MP.6**	Coherence KY.5.MD.5→KY.6.G.2→KY.7.G.6
KY.6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **MP.4**, **MP.5**, **MP.6**	For example, a gardener draws a map of his garden on a coordinate plane with vertices (-2, 7), (-2, -1), (4, 7). What is the base and height of this triangle? What is the area of his garden, assuming each unit on the coordinate plane is 1 meter? Coherence KY.5.G.2→KY.6.G.3
KY.6.G.4 Classify three-dimensional figures including cubes, prisms, pyramids, cones and spheres. **MP.2**, **MP.3**	Emphasis is on classifying three-dimensional shapes and specifically the attributes of each shape that make it unique to its classification. Coherence KY.6.G.4→KY.7.G.6

Attending to the Standards for Mathematical Practice
Students make sense of real-world problems involving area, volume and surface area. Students begin to understand any shape can be thought of as a series of simpler shapes, merely stitched together to form a composite shape (**MP.1**). They begin to visualize the volume of any given shape as a bounded region, filled with smaller cubes of equal size (**MP.2**) and understand, by doing so, they approximate the volume of a three-dimensional shape with integer edge lengths (**MP.5**) and then, continue this reasoning by precisely finding the volume of figures with rational edge lengths (**MP.1**, **MP.6**, **MP.8**). Generalizing the study of geometric shapes to the coordinate plane gives students a tool to precisely calculate side lengths and area of shapes. When two different units are given within a problem, students know to use previous knowledge of conversions to make the units match before solving the problem (**MP.4**, **MP.5**, **MP.6**).

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Develop understanding of the process of statistical reasoning.	
Standards	**Clarifications/Illustrations**
KY.6.SP.0 Apply the four-step investigative process for statistical reasoning. a. Formulate Questions: Formulate a statistical question as one that anticipates variability and can be answered with data. b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual and comparing individual to group. **MP.1**, **MP.4**	Emphasis is on understanding answering a statistical question is completed by an investigative process that encompasses questioning, collection, analysis and interpretation of the data gathered. Coherence KY.5.MD.2→KY.6.SP.0→KY.7.SP.1
Attending to the Standards for Mathematical Practice	
The four-step investigative process provides a structure for students to follow that allows them to model many real-world situations with a model (**MP.4**). Students use the statistical process to seek to understand the world around them, taking time to pursue the entire process in order to gain insights, looping back to make revisions to the question or data gathering if the results they have do not adequately address their question (**MP.1**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Develop understanding of statistical variability.	
Standards	**Clarifications**
KY.6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. **MP.1**, **MP.3**, **MP.6**	For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates a variety of values with associated variability in students' ages. Coherence KY.5.MD.2→KY.6.SP.1→KY.7.SP.1
KY.6.SP.2 Understand that a set of numerical data collected to answer a statistical question has a distribution which can be described by its center, spread and overall shape. **MP.2**, **MP.6**, **MP.7**	Students distinguish between graphical representations which are skewed or approximately symmetric; use a measure of center to describe a set of data. Coherence KY.5.MD.2→KY.6.SP.2→KY.7.SP.3
KY.6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number to describe a typical value, while a measure of variation describes how the values in the distribution vary. **MP.2**, **MP.5**, **MP.6**	Emphasis is on the sensitivity of measures of center to changes in the data, such as mean is generally much more likely to be pulled towards an extreme value than the median. Additionally, measures of variation (range, interquartile range) describe the data by giving a sense of the spread of data points. Coherence KY.6.SP.3→KY.7.SP.4

Attending to the Standards for Mathematical Practice
Students recognize a question such as "What did I eat for breakfast?" is not a statistical question, whereas "What is the most popular breakfast in my school?" will elicit data they can measure precisely (**MP.6**) and draw conclusions based on that data (**MP.3**). After collecting data, by creating a distribution of that data, students recognize data generally follows a structure and can be described in terms of that structure (**MP.7**). By accurately calculating the mean (or any other statistical measure), students are now more precise in describing data, going from, for example, describe the rainfall for the month as "about average" to "the rainfall this month is slightly higher than the mean of the last 10 years and within the interquartile range for that data." (**MP.6**)

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Summarize and describe distributions.	
Standards	**Clarifications**
KY.6.SP.4 Display the distribution of numerical data in plots on a number line, including dot plots, histograms and box plots. **MP.6**, **MP.7**	Students create the listed graphical representations in the appropriate context and describe the attributes of each. Coherence KY.5.MD.2→KY.6.SP.4→KY.7.SP.1
KY.6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Determining quantitative measures of center (median and/or mean) to describe distribution of numerical data. d. Describing distributions of numerical data qualitatively relating to shape (using terms such as cluster, mode(s), gap, symmetric, uniform, skewed-left, skewed-right and the presence of outliers) and quantitatively relating to spread/variability (using terms such as range and interquartile range). e. Relating the choice of measures of center and variability to the shape of the data distribution. **MP.3**, **MP.7**	a. Students understand larger numbers of observations create a more accurate statistical representation than smaller numbers of observations. b. Students describe how the data measured relates to answering a statistical question. c. Students know methods of finding measures of center, including finding median in non-ordered sets of data and a mean is a mathematical average. d. Students describe the shape of data by inspection using the terms listed and calculate the range and interquartile range of a set of data. e. Students recognize mean and range are appropriate measures for symmetrical data while the median and interquartile range may be better measures for skewed data. Coherence KY.6.SP.5→KY.7.SP.1
Attending to the Standards for Mathematical Practice	
Students make use of structure by aligning numerical data into plots and histograms. Students characterize their data in a distribution using mathematically precise terms, both quantitatively (mean, IQR, etc.) and qualitatively (skewed, clustered, etc.). (**MP.7**). Students summarize their data in a variety of ways, both numerically and graphically and use these summaries to draw conclusions about their results (**MP.3**). Additionally, because students are calculating precisely the measures of center and variability for their data, they accurately compare data sets in a variety of ways (**MP.6**).	

Kentucky Academic Standards for Mathematics: Grade 7 Overview

Ratio and Proportional Relationships (RP)	The Number System (NS)	Expressions and Equations (EE)	Geometry (G)	Statistics and Probability (SP)
• Analyze proportional relationships and use them to solve real-world and mathematical problems.	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.	• Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area and volume.	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use and evaluate probability models.

In grade 7, instructional time should focus on four critical areas:

1. **In the Ratios and Proportional Relationships domain, students will:**
    - extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems;
    - use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips and percent increase or decrease;
    - solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects;
    - graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope;
    - distinguish proportional relationships from other relationships.

2. **In the Number System and the Expressions, Equations and Inequalities domains, students will:**
    - develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation) and percents as different representations of rational numbers;
    - extend addition, subtraction, multiplication and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division--by applying these properties and by viewing negative numbers in terms of everyday contexts;
    - explain and interpret the rules for adding, subtracting, multiplying and dividing with negative numbers;
    - use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. **In the Geometry domain, students will:**
    - continue their work with area from grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects;

- reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions and they gain familiarity with the relationships between angles formed by intersecting lines;
- work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections;
- solve real-world and mathematical problems involving area, surface area and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

2. In the Statistics and Probability domain, students will:

- build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations;
- begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Ratios and Proportional Relationships	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Analyze proportional relationships and use them to solve real-world and mathematical problems.**	

Standards	Clarifications
KY.7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. **MP.2**, **MP.6**	For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour. KY.6.RP.2 Coherence KY.6.RP.3→ KY.7.RP.1
KY.7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities represent a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **MP.1**, **MP.2**, **MP.3**	a. Students test for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Students understand finding the unit rate in a table or graph is equivalent to the constant of proportionality in an equation or verbal description. KY.8.F.2 KY.8.F.4 Coherence KY.6.RP.3a →KY.7.RP.2b →KY.8.EE.6 c. If total cost *t* is proportional to the number n of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$. Coherence KY.7.RP.2c → KY.8.EE.5 d. Students describe points (*x, y*) in terms of the labels of the *x*- and *y*-axes; students understand in a proportional relationship (0, 0) is a valid point and (1, *r*) represents the unit rate and the constant of proportionality for the relationship between the quantities. Coherence KY.7.RP.2d → KY.8.F.5

KY.7.RP.3 Use percents to solve mathematical and real-world problems. a. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, a part and a percent, given two of these. b. Use proportional relationships to solve multistep ratio and percent problems. **MP.5**, **MP.6**	a. For example, 30% of a quantity means 30/100 times the quantity. b. Could include but not limited to simple interest, tax, markups and markdowns, gratuities and commissions, percent increase and decrease, percent error. Coherence KY.6.RP.3c → KY.7.RP.3

Attending to the Standards for Mathematical Practice
Translating a rate to a unit rate allows students to contextualize a complex ratio to something more likely for them to understand, for example, a rate of miles per ONE hour or gallons per ONE minute (**MP.2**). The use of unit rates allows students to be precise in their understanding, transferring "½ mile in ¼ hour" to something understandable, such as 2 miles per hour (**MP.1**). Students think about why some relationships are proportional where others are not. Students make sense of and solve multistep ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane and equations and relate these representations to each other and to the context of the problem. Students depict the meaning of the constant of proportionality in proportional relationships and the importance of (0, 0) and (1, *r*) on graphs (**MP.1**). Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Students use concrete numbers to create and implement equations, including $y = kx$, where *k* is the constant of proportionality. (**MP.2**) One special proportional relationship in common usage involves percents. Students may think about "percent" as "part of 100" and solve a proportional relationship for any missing part of the relationship between a number, a part of that number and the associated percentage (**MP.5**). Students reason about when their resulting solutions make sense, as when the resulting solution is greater than 100% or, when speaking about percent increase, decrease and error, when their resulting solution may be a negative value (**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

<table>
<tr><th colspan="2">The Number System</th></tr>
<tr><th colspan="2">Standards for Mathematical Practice</th></tr>
<tr><td>MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.</td><td>MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.</td></tr>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0.
b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.
MP.2, MP.4, MP.7</td><td>a. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. The sum of numbers is a directional movement from one number to another for a specified amount of spaces on the number line. The sum of opposites is 0 due to the fact that opposites have equivalent absolute values.
c. Subtracting a positive number is the same as adding the positive number’s opposite.

KY.6.NS.5
KY.6.NS.6
Coherence KY.6.NS.7→ KY.7.NS.1</td></tr>
<tr><td>KY.7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules</td><td>a. Emphasis is on exploring and understanding how the rules for multiplying and dividing with negative numbers are connected to properties for the operations, rather than to think of them as arbitrary rules. They explain 4 times (-3) could be four days of golfing 3 under par and therefore, having an overall score of -12. The remaining operations are based on applying properties.</td></tr>
</table>

for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. **MP.2**, **MP.7**, **MP.8**	b. Emphasis is on the equivalence relationship provided by the movement of one negative sign among the numerator, denominator, or in front of the entire fraction. Coherence KY.6.NS.1→ KY.7.NS.2 → KY.8.NS.1
KY.7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. **MP.1**, **MP.2**, **MP.5**	Emphasis is on applying mathematical operations to rational numbers that occur in real world context. Coherence KY.6.NS.3 → KY.7.NS.3

Attending to the Standards for Mathematical Practice
In grade 7, students build upon understanding by examining inverses and reason any number has an additive inverse, which is the mirror-image of the original number, albeit on the opposite side of zero, which brings the idea of absolute value to life (**MP.2**). The structure of working with the various properties of rational numbers cannot be ignored and students systematically apply these properties in a variety of scenarios (**MP.7**). Understanding these properties gives students a tool to model many real-world situations with simpler mathematical sentences. Through the use of number lines, tape diagrams, expressions and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples (**MP.4**). Students demonstrate fluency in applying the four operations to rational numbers in real life situations when they strategically apply the properties of operations to model real-world situations and truly making sense of the world around them with mathematics. Additionally, as students fluently solve word problems, they consider their steps and determine whether or not they make sense in relationship to the arithmetic understanding that served as their foundation in earlier grades (**MP.1**, **MP.2**, **MP.4**, **MP.5**). Students move from recall of applying rules of multiplying and dividing signed numbers to the ability to apply these rules strategically in a variety of situations. Students formulate rules for operations with signed numbers by observing patterns (**MP.2**, **MP.8**).

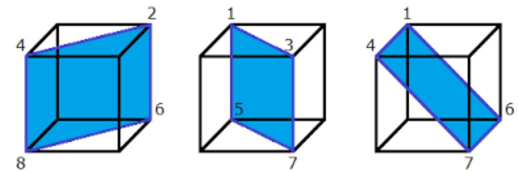
Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use properties of operations to generate equivalent expressions.	
Standards	**Clarifications**
KY.7.EE.1 Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. **MP.2**, **MP.3**	Students demonstrate understanding of applying the order of operations to an expression involving multiple operations, including using the distributive property and variables in the expression. Students apply the properties of commutative, associative and distributive fluently. Coherence KY.6.EE.3 → KY.7.EE.1 → KY.8.EE.7
KY.7.EE.2 Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. **MP.7**, **MP.8**	Students apply mathematical properties in order to rewrite expressions and clarify the relationship of quantities in a problem. For Example: If Tom and Jim both get paid a wage of $11 per hour, but Tom was paid an additional $55 for overtime, the expression $11(T + J) + 55$ may be more clearly interpreted as $11T + 55 + 11J$ for purposes of understanding Tom's pay separated from Jim's pay. Coherence KY.6.EE.4 → KY.7.EE.2 → KY.8.EE.8c
Attending to the Standards for Mathematical Practice	
Students who fluently use the strategies of the properties of rational numbers to reason through the standard order of operations by applying these properties in a structured way. Students recognize the repeated use of the distributive property as they write equivalent expressions (**MP.7**). When given an example problem involving multiple operations containing a mistake, students answer the question "Where did the mistake occur and how do I know?" (**MP.3**). Students bring mathematical context to real-life situations by understanding multiple representations of quantities may exist. For example, adding 5% to quantity *a* leads to an expression of *a + .05a = 1.05a*, which clarifies the problem. Students access previous knowledge of working with percents to use the same structure to see equivalent expressions exist, even when taken out of the context of the real-world situation (**MP.7**). Students extend this reasoning to understand other situations (**MP.8**).	

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**	
Standards	**Clarifications**
KY.7.EE.3 Solve real-life and mathematical problems posed with positive and negative rational numbers in any form, using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. **MP.1**, **MP.4**, **MP.6**	Students solve multi-step real-world and mathematical problems containing integers, fractions and decimals, using previously acquired skills around converting fractions, decimals and percentages and use properties of operations to find equivalent forms of expressions when needed. Students solidify understanding by checking their solutions for reasonableness using estimation strategies such as rounding, compatible numbers and benchmark numbers. Coherence KY.7.NS.3 →KY.8.EE.4
KY.7.EE.4 Use variables to represent quantities in a real-world or mathematical problem and construct equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q* and *r* are specific rational numbers. Solve equations of these forms. Graph the solution set of the equality and interpret it in context of the problem. b. Solve word problems leading to inequalities of the form $px + q > r$, $px + q < r$, $px + q \geq r$, $px + q \leq r$; where *p*, *q* and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in context of the problem. **MP.2**, **MP.4**	a. Interpret word problems in the form of the initial value as a one-time occurrence within the problem and the coefficient as the recurring event within the problem. Coherence KY.6.EE.7 → KY.7.EE.4 → KY.8.EE.7 b. Interpret word problems having one or more solutions that satisfy the conditions of the problem. Graph on a number line the solution set that satisfies the conditions of the problems. Coherence KY.6.EE.8 → KY.7.EE.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
It is common for students to have difficulty in scaffolding from simple problems to more complex, multi-step problems; assistance in this regard is given by the use of estimation strategies to benchmark their work and lend confidence to more accurate solutions (**MP.1**, **MP.6**). Students apply the properties of rational numbers in order to solve equations and inequalities. Students must be precise when defining a variable (**MP.6**). Students reason a solution to a real-life situation but may struggle with modeling the problems with an equation or inequality involving a variable. For example, "I buy 6 pencils and a $3 pen for a total of $12. How much did each pencil cost?" Students with an understanding of numbers, but not the idea of a variable, may create an equation of $p = \frac{12-3}{6}$ = 1.50. Students who successfully model with mathematics understand the variable represents the cost of one pencil and use it appropriately, $6p + 3 = 12$, which more accurately represents the problem presented (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Draw**, **construct and describe geometrical figures and describe the relationships between them.**	
Standards	**Clarifications**
KY.7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **MP.1**, **MP.2**, **MP.5**	Emphasis is on being able to convert values from one given measurement to another based on a given scale factor. For example, 1 inch on the scale drawing equals how many feet in real life based on the scale factor given. Students reproduce a given drawing based on a scale factor. Coherence KY.6.G.1→KY.7.G.1→KY.8.EE.6
KY.7.G.2 Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **MP.6**, **MP.7**	Emphasis is on taking given conditions and converting them to geometric shapes, constructing triangles with given angle measures and side lengths and determining when the given conditions do not meet the conditions of a triangle. Coherence KY.7.G.2→KY.8.G.1
KY.7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **MP.5**, **MP.6**	Cross sections may be taken from horizontal, vertical and oblique angles, such as: 

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students extend their knowledge of proportional reasoning to solve problems involving dimensions and area. Proper use of tools help them understand the conditions by which three side lengths will determine one triangle or no triangle. Students have opportunities to reflect on the appropriateness of a tool for a particular task (**MP.5**). Initially, students may struggle with moving from a concrete understanding of a real-world situation to a miniature version, or vice versa; hands-on measurements and the use of technology can assist students with this abstract idea. In many cases, students make sense of new and different contexts and engage in significant struggle to solve problems (**MP.1**, **MP.2**). Students begin to understand it may not be possible to draw a certain shape with given measurements, or, if possible, may not yield a unique shape and reason why this may be the case (**MP.7**). By finding the constraints that exist in the Triangle Inequality Theorem, for example, a student determines precisely when a triangle may or may not exist (**MP.6**). By emphasizing the differences in various slicing planes, students accurately represent the resulting sections (**MP.6**).

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-life and mathematical problems involving angle measure, area, surface area and volume.**	
Standards	**Clarifications**
KY.7.G.4 Use formulas for area and circumference of circles and their relationships. a. Apply the formulas for the area and circumference of a circle to solve real-world and mathematical problems. b. Explore and understand the relationship between the radius, diameter, circumference and area of a circle. **MP.1**, **MP.2**, **MP.8**	Circle Formulas: C=dπ $C = 2r\pi$ A=πr^2 Note: Calculating the radius or diameter of a circle given its area is not expected, as finding the square root of a number is reserved for 8th grade. a. Both area and circumference are represented; students recognize when circumference is needed and when area is needed. b. Emphasis is on calculating area given diameter; finding circumference given radius or diameter; and finding radius or diameter given circumference. Special attention given to the relationship between diameter and circumference as a ratio that leads to pi. Coherence KY.7.G.4 → KY.8.G.9
KY.7.G.5 Apply properties of supplementary, complementary, vertical and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **MP.3**, **MP.6**, **MP.7**	Emphasis is on the relationships between the various angles listed to find missing angles based on the relationships and to write and solve equations to find unknown angles. KY.8.G.1 Coherence KY.4.MD.7 →KY.7.G.5 →KY.8.G.5
KY.7.G.6 Solve problems involving area of two-dimensional objects and surface area and volume of three-dimensional objects. a. Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles, quadrilaterals and other polygons.	a. Emphasis is on finding the area of composite figures composed of convex polygons. b. Students understand volume and surface area are two different quantities used to describe the same three-dimensional figure. Building upon their understanding of area, students use nets of three dimensional objects to conceptualize surface area. Students calculate with appropriate units, using nets as a

b. Solve real-world and mathematical problems involving volume and surface area, using nets as needed, of three-dimensional objects including cubes, pyramids and right prisms. **MP.3**, **MP.4**, **MP.5**	possible strategy for calculation as well as formulas for volume and surface area, where appropriate. KY.6.G.1 KY.6.G.2 Coherence KY.6.G.4→ KY.7.G.6 → KY.8.G.6

Attending to the Standards for Mathematical Practice
A student who merely memorizes the area and circumference formulas for a circle or the area, volume and surface area formulas of other shapes does not have a deep, conceptual understanding of the basis for these equations. Exploring the relationships between radius, diameter, area and circumference limits the confusion inherent in rote memorization, because students are given a context to the concepts (**MP.2**, **MP.8**). Solving real-world situations involving these quantities gives the student context for their understanding of the mathematics (**MP.1**). In addition, precise drawing or manipulation of technology lends itself to generate definitions (**MP.6**). Students continue their work from grade 6 from solving area problems involving triangles and rectangles to those involving more complex shapes, such as rhombi or trapezoids (**MP.4**). Students may mischaracterize volume and surface area of three dimensional shapes, leading them to develop ways to decide upon whether a situation calls for the volume of a figure, or the surface area of a figure (**MP.3**). The use of nets and other appropriate tools gives students a structure to foster greater understanding of the concept of surface area (**MP.5**).

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use random sampling to draw inferences about a population.	
Standards	**Clarifications**
KY.7.SP.0 Create displays, including circle graphs (pie charts), scaled pictographs and bar graphs, to compare and analyze distributions of categorical data from both matching and different-sized samples. **MP.2**, **MP.3**, **MP.6**	Students have been introduced to pictographs and bar graphs in grades 2 and 3; Circle graphs are new and connect to the grade 7 focus on percents. Also, students' knowledge of rates mean they can approach scaled pictographs in a more sophisticated manner. An important aspect of doing statistics is *selecting* an appropriate data display for the question under investigation. Students need to be asked, "Which data display fits this data set and why?" The circle graph focuses more on the relative values of the clustering of data, whereas the bar and pictographs add a dimension of quantity. The choice of which data display (and how categories are set up within each display) will result in different pictures of the shape of data. Finally students are comparing two distributions. When comparing two different distributions, circle graphs lend to comparing different sized samples, because circle graphs are based on percentages. KY.7.SP.0 KY.7.SP.2 Coherence KY.6.SP.O→KY.7.SP.4
KY.7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	Recognize what makes a valid and non-valid sample of a population. Recognize the size of the sample holds importance to the accuracy of the sample. KY.6.SP.0 KY.6.SP.1 Coherence KY.6.SP.2→KY.7.SP.1→KY.HS.SP.9

MP.3, **MP.6**	
KY.7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. a. Generate multiple samples of categorical data of the same size to gauge the variation in estimates or predictions. b. Generate multiple samples (or simulated samples) of numerical data to gauge the variation in estimates or predictions. c. Gauge how far off an estimate or prediction might be related to a population character of interest. **MP.2**, **MP.3**, **MP.7**	Emphasis is on the sample size and how this affects the validity of the estimate or prediction. Examples: a. Randomly sample 6th, 7th and 8th graders about who their favorite superhero is to generate samples of data that are roughly the same size, looking specifically at patterns, if any. b. Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Coherence KY.6.SP.0→ 7.SP.2→ KY.HS.SP.12

Attending to the Standards for Mathematical Practice
Students understand the method of sampling a population affects the reliability and validity of the data gleaned, so they justify their conclusions and inferences in a valid way (**MP.3**). In doing so, they create an accurate picture of the question posed (**MP.6**). In drawing inferences and reasoning about the variation of their estimates, students construct arguments based on data (**MP.2**, **MP.3**). When students, for example, examine a sample of 10 data points, versus a sample of 100 data points, they generalize why the samples may have two different sample errors (**MP.7**).

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Draw informal comparative inferences about two populations.**	
Standards	**Clarifications**
KY.7.SP.3 Describe the degree of visual overlap (and separation) from the graphical representations of two numerical data distributions (box plots, dot plots) with similar variabilities with similar contexts (same variable), measuring the difference between the centers (medians or means) by expressing this difference as a multiple of a measure of variability (interquartile range when comparing medians or the mean absolute deviation when comparing means). **MP.1**, **MP.5**, **MP.7**	For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. KY.6.SP.2 Coherence KY.6.NS.1→KY.7.SP.3→KY.HS.SP.13 KY.HS.SP.10
KY.7.SP.4 Calculate and use measures of center (mean and median) and measures of variability (interquartile range when comparing medians and mean absolute deviation when comparing means) for numerical data from random samples to draw informal comparative inferences about two populations. **MP.2**, **MP.5**, **MP.7**	For example, decide whether the words in a chapter of a grade seven science book are generally longer than the words in a chapter of a grade four science book. KY.HS.SP.10 Coherence KY.6.SP.2→KY.7.SP.4→KY.HS.SP.13

Attending to the Standards for Mathematical Practice
When comparing two data distributions, students visually note differences, for example, of two dot plots. What is more difficult at times is to conceptualize this in mathematical terms, such that one distribution may have twice the variability of the other (**MP.2**). In moving from visual representation to measures of center and variability, students using these measures mathematically describe a situation that may be difficult to otherwise describe (**MP.5**, **MP.7**). Categorically summarizing data in circle graphs, gives students a basis for bringing their number sense from percents to statistics, allowing them to be precise when describing data (57% of students have brown shoes) (**MP.6**), while reasoning and drawing conclusions from data presented (**MP.2**, **MP.3**). Now, students drawing inferences from their calculations they have learned in grade 6 and earlier in grade 7 allows them to use these tools (**MP.5**) and allows them to mathematically compare (**MP.7**) in such a way that their inferences and conclusions make sense in context (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Investigate chance processes and develop**, **use and evaluate probability models.**	
Standards	**Clarifications**
KY.7.SP.5 Describe the probability of a chance event is a number between 0 and 1, which tells how likely the event is, from impossible (0) to certain (1). A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event. **MP.5**, **MP.6**, **MP.7**	Emphasis is on descriptive language used to describe numerical probabilities; impossible event, unlikely event, equally likely event, likely event, certain event. Students understand all probabilities must fall between 0 and 1.
KY.7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability. **MP.1**, **MP.2**	Estimate the likelihood of an event, test the estimate by trial and collect data. Students observe accuracy of the estimate will increase with the frequency of repeated trials. Coherence KY.7.SP.6→ KY.HS.SP.10
KY.7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. **MP.4**, **MP.7**, **MP.8**	For example: a. If a student is selected at random from a class, find the probability Jane will be selected and the probability a girl will be selected. b. Find the approximate probability a spinning penny will land heads up or a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? KY.7.RP.3 Coherence KY.7.SP.7→ KY.HS.SP.14
KY.7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.	Example: a. If the probability of heads occurring on a coin is ½, then the probability of three heads in a row is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

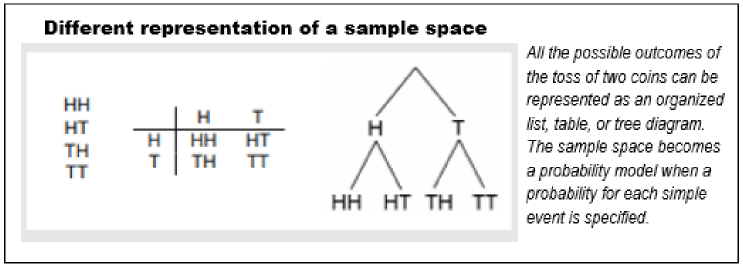
a. Explain just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events described in everyday language using methods such as organized lists, tables and tree diagrams.

c. Design and use a simulation to generate frequencies for compound events.

MP.2, **MP.4**, **MP.7**

b. For a simulation of tossing two fair coins:



c. Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability it will take at least 4 donors to find one with type A blood?

Coherence KY.7.SP.8→ KY.HS.SP.14

Attending to the Standards for Mathematical Practice

Thinking of probability as being on a continuum ranging from a probability of 0 to a probability of 1 allows students to visualize the structure of ranking the chances of an event occurring (**MP.7**). When they relate these broader terms to actual calculated probability, this lends precision to otherwise vague concepts (**MP.6**). In addition, students note the opposite is also true; a calculated probability close to ½ means the event is neither unlikely nor likely, or equally likely (**MP.5**). Looking at the process that generates a set of probabilities (experimental probability) in a specific scenario gives students the opportunity to examine a situation in depth (**MP.1**) and reason about why the conclusion they draw may or may not be accurate (**MP.2**). Student thinking about theoretical probability is extended to developing a model (**MP.4**) that lends structure (**MP.7**) to an otherwise abstract idea. Students may use this model to explain why a penny comes up heads half the time and tails the other half, but in an experiment where this event is repeated multiple times, the experimental probability may not be exactly ½ and ½. (**MP.8**). Compound probability may be more difficult for students to understand; tree diagrams, lists, etc. may help students understand the concept (**MP.7**). Difficult to understand compound events may necessitate a simulation tool, for example a random digit generator (**MP.4**).

Kentucky Academic Standards for Mathematics: Grade 8 Overview

The Number System (NS)	Expressions and Equations (EE)	Functions (F)	Geometry (G)	Statistics and Probability (SP)
• Know that there are numbers that are not rational and approximate them by rational numbers.	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.	• Define, evaluate and compare functions. • Use functions to model relationships between quantities.	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.	• Investigate patterns of association in bivariate data.

In grade 8, instructional time should focus on three critical areas:

1. **In the Number System, the Expressions, Equations and Inequalities, and the Probability and Statistics domains, students will:**
    - recognize equations for proportions (y/x = m or y=mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope and the graphs are lines throughout the origin;
    - understand that the slope (m) of a line is a constant rate of change, as well as how the input and output changes as a result of the constant rate of change;
    - interpret a model in the context of the data by expressing a linear relationship between the two quantities in question and interpret components of the relationship (such as slope and y-intercept) in terms of the situation;
    - solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line;
    - use linear equations, systems of linear equations, linear functions and their understanding of slope of a line to represent, analyze and solve a variety of problems.
2. **In the Functions and the Expressions, Equations and Inequalities domains, students will:**
    - grasp the concept of a function as a rule that assigns to each input exactly one output;
    - understand that functions describe situations where one quantity determines another;
    - translate among representations and partial representations of functions (nothing that tabular and graphical representations may be partial representations of the function) and describe how aspects of the function are reflected in the different representations.
3. **In the Geometry domain, students will:**
    - use ideas about distance and angles, how they behave under translations, rotations, reflections and dilations and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems;
    - show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines;
    - understand the statement of the Pythagorean Theorem and its converse, and why the Pythagorean Theorem holds;
    - apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths and to analyze polygons.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Know that there are numbers that are not rational and approximate them by rational numbers.**	
Standards	**Clarifications**
KY.8.NS.1 Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational. **MP.2**, **MP.6**, **MP.7**	Emphasis is placed on how all rational numbers can be written as an equivalent decimal. The end behavior of the decimal determines the classification of the number. Coherence KY.7.NS.2→ KY.8.NS.1→ KY.HS.N.3
KY.8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions. **MP.2**, **MP.7**, **MP.8**	For example, by shortening the decimal expansion of $\sqrt{2}$ by dropping all decimals past a certain point and showing $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5 and so on. Students recognize this process could be repeated an infinite number of times. Coherence KY.8.NS.2→ KY.HS.N.3

Attending to the Standards for Mathematical Practice

Students attend to precision (**MP.6**) by recognizing and identifying numbers as rational or irrational. Students know the definition of an irrational number and represent the number in different ways, as a root, non-repeating decimal block, or symbol. Students attend to precision when clarifying the difference between an exact value of an irrational number compared to the decimal approximation of the irrational number. Ultimately, students come to an informal understanding (**MP.2**) the set of real numbers consists of rational numbers and irrational numbers. They continue to work with irrational numbers and rational approximations when solving equations such as $x^2 = 18$. While using the long division algorithm to convert fractions to decimals, students recognize when a sequence of remainders repeats that the decimal form of the number will contain a repeat block (**MP.8**). Students recognize when the decimal expansion of a number does not repeat or terminate, the number is irrational and can be represented with a method of rational approximation using a sequence of rational numbers to get closer and closer to the given number (**MP.7**). Students look for structure in repeating decimals, recognize repeating blocks and know every fraction is equal to a repeating decimal.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Work with radicals and integer exponents.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
MP.3, MP.7, MP.8</td><td>
<table>
<tr><th>Name</th><th>Product of Powers</th><th>Quotient of Powers</th><th>Power of a Product</th><th>Power of a Quotient</th><th>Power of a Power</th><th>Negative Exponent</th></tr>
<tr><td>Property</td><td>$a^m \cdot a^n = a^{m+n}$</td><td>$\frac{a^m}{a^n} = a^{m-n}$</td><td>$(a \cdot b)^n = a^n \cdot b^n$</td><td>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$</td><td>$(a^m)^n = a^{mn}$</td><td>$a^{-n} = \frac{1}{a^n}$</td></tr>
</table>
Coherence KY.8.EE.1→ KY.HS.N.1</td></tr>
<tr><td>KY.8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that perfect squares and perfect cubes are rational.
MP.5, MP.6</td><td>Students do not prove these are the only solutions, but rather use informal methods, such as guess and check. For example, $\sqrt{64} = \sqrt{8^2} = 8$ and $\sqrt[3]{5^3} = 5$. Since $\sqrt{p}$ is defined to mean the positive solution to the equation $x^2 = p$ (when it exists), it is not correct to say (as is common) $\sqrt{64} = \pm 8$. In describing the solutions to $x^2 = 64$, students write $x = \pm\sqrt{64} = \pm 8$.
Coherence KY.8.EE.2→ KY.HS.A.12</td></tr>
<tr><td>KY.8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 (Scientific Notation) to estimate very large or very small quantities and express how many times larger or smaller one is than the other.
MP.3, MP.5, MP.6</td><td>Students conceptualize why a number could be written in scientific notation and the benefits of doing so and connect exponent rules learned earlier to the methods of writing a quantity in scientific notation.
Coherence KY.8.EE.3→ KY.HS.N.6</td></tr>
<tr><td>KY.8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.
MP.2, MP.5, MP.6</td><td>Choose appropriate units for real-life situations. When solving problems and using technology, it is possible solutions are given that take the form of $1.2\ x\ 10^{00}$ or $3.4\ x\ 10^{-07}$. Some technologies also use a capital E when denoting numbers such a $1.45E07$ or $4.665E$-11.
Coherence KY.8.EE.4→ KY.HS.N.4</td></tr>
</table>

Attending to the Standards for Mathematical Practice
Students construct mathematical arguments and reasoning emphasized as students learn the properties of exponents (**MP.3**). Students reason $5^3 \cdot 5^2 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5) = 5^5$ through numerous experiences of working with exponents, students generalize the properties of exponents (**MP.7**) before using them fluently. Students notice if calculations are repeated (**MP.8**) and look both for general methods and for shortcuts. Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used (**MP.2**, **MP.7**, **MP.8**). Students compare and interpret scientific notation quantities in the context of the situation, recognizing the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand the connections between proportional relationships, lines and linear equations.**	
Standards	**Clarifications**
KY.8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **MP.2**, **MP.3**, **MP.4**	Emphasis is on relating previous knowledge of unit rate to slope in tables, graphs, equations and sets of ordered pairs and comparing the slopes of two different proportional relationships. Different ways the proportional relationships can be represented include tables, graphs, equations, or sets of ordered pairs. KY.8.F.2 Coherence KY.7.RP.2→ KY.8.EE.5→ KY.HS.A.23
KY.8.EE.6 Use similar triangles to explain why the slope, *m*, is the same between any two distinct points on a non-vertical line in the coordinate plane; know the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*. **MP.3**, **MP.4**, **MP.7**	Using the properties of similar triangles, demonstrate the slope between any two pairs of points on a non-vertical line create the same rise-run ratio when simplified. Understand $y = mx$ and $y = mx + b$ differ in that $y = mx$ only has the possibility of 0 being the *y*-intercept and that $y = mx + b$ has infinite possibilities, including 0, for the *y*-intercept depending on the value of *b*. KY.HS.G.22 Coherence KY.7.RP.2→ KY.8.EE.6→ KY.HS.A.23
Attending to the Standards for Mathematical Practice	
Students represent real-world situations symbolically (**MP.4**). Students identify important quantities from a context and represent the relationship in the form of an equation, a table and a graph. Students analyze the various representations and draw conclusions and/or make predictions (**MP.3**). Once a solution or prediction has been made, students reflect on whether the solution makes sense in the context presented (**MP.4**). One example of this is when students determine how many buses are needed for a field trip. As this is most probably not an exact solution, students must interpret their fractional solution and make sense of it as it applies to the real world. Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. Students use the structure of an equation to make sense of the information in the equation (**MP.7**). For example, students write equations that represent the constant rate of motion for a person walking. In doing so, they interpret an equation such as $y = \frac{3}{5}x$ as the total distance a person walks, y, in x	

amount of time, at a rate of $\frac{3}{5}$. Students look for patterns or structure in tables and show a rate is constant; students also understand a lack of a pattern represents a non-constant (non-linear) rate.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Analyze and solve linear equations and pairs of simultaneous linear equations.**	
Standards	**Clarifications**
KY.8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms. **MP.2**, **MP.3**, **MP.7**	Building upon skills from grade 7, students combine like terms on the same side of the equality and use the distributive property to simplify the equation when solving. Emphasis in this standard is also on using rational number coefficients. Solutions of certain equations may elicit infinitely many or no solutions. Coherence KY.7.EE.1→ KY.8.EE.7→ KY.HS.A.18
KY.8.EE.8 Analyze and solve a system of two linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously; understand that a system of two linear equations may have one solution, no solution, or infinitely many solutions. b. Solve systems of two linear equations in two variables algebraically by using substitution where at least one equation contains at least one variable whose coefficient is 1 and by inspection for simple cases c. Solve real-world and mathematical problems leading to two linear equations in two variables. **MP.1**, **MP.3**, **MP.4**	a. Examples are both mathematical and real-life contexts. Emphasis is on determining what types of contexts lead to having no solutions or infinitely many solutions. Students use tables, graphs and equations to explain why a graphed system has infinitely many or no solutions. b. Elimination and/or matrices are not required for grade 8. Emphasis is on *choosing* a method. Students solve simple cases by inspection, for example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6 and select from the other approaches, based on the numbers in the problem. Solving systems algebraically will be limited to at least one equation containing at least one variable with a coefficient of 1; for example, $y = 3x$,

	$y = -12x + 6, x = 2, x = 2y + 1.$ Coherence KY.7.EE.2→ KY.8.EE.8→ KY.HS.A.20

Attending to the Standards for Mathematical Practice
Students solve linear equations in one variable, including cases with one solution, an infinite number of solutions and no solutions. Students show examples of each of these cases by successively transforming an equation into simpler forms. Some linear equations require students to expand expressions by using the distributive property and to collect like terms (**MP.2**, **MP.7**). Solving pairs of simultaneous linear equations builds on the skills and understandings students used to solve linear equations with one variable and systems of linear equations may also have one solution, an infinite number of solutions, or no solutions (**MP.2**, **MP.3**). Students discover these cases as they graph systems of linear equations and solve algebraically.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Define**, **evaluate and compare functions.**	

Standards	Clarifications
KY.8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **MP.7**, **MP.8**	Students understand the reasoning that not all relations are functions. Note: Function notation is not required in grade 8. Coherence KY.8.F.1→ KY.HS.F.1
KY.8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **MP.1**, **MP.2**, **MP.4**	Given a linear function represented using one method listed and another linear function represented by different method listed, determine which function has the greater or lesser rate of change or greater or lesser initial value. Coherence KY.7.RP.2→ KY.8.F.2→ KY.HS.F.1
KY.8.F.3 Understand properties of linear functions. a. Interpret the equation *y* = *mx* + *b* as defining a linear function, whose graph is a straight line. b. Identify and give examples of functions that are not linear. **MP.7**	a. For example, the equation $c = 3g + 5$ models the linear function for the total cost, *c*, of bowling, where *g* represents the number of games played and shoe rental is $5. b. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. Coherence KY.7.EE.4→ KY.8.F.3→ KY.HS.F.11

Attending to the Standards for Mathematical Practice
Students examine, interpret and represent functions symbolically (**MP.2**, **MP.4**). They make sense of quantities and their relationships in problem situations (**MP.2**). For example, students make sense of values as they relate to the total cost of items purchased or a phone bill based on usage in a particular time interval. Students use what they know about rate of change to distinguish between linear and nonlinear functions (**MP.8**). Further, students contextualize information gained from the comparison of two functions (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use functions to model relationships between quantities.	
Standards	**Clarifications**
KY.8.F.4 Construct a function to model a linear relationship between two quantities. a. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. b. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. **MP.4**, **MP.5**, **MP.8**	Examining a relationship between two quantities yields a function rule. This function rule can be described using its initial value and rate of change, from a variety of representations, including tables, graphs, equations and verbal descriptions. Understand the rate of change and initial value in terms of the situation it models. KY.HS.F.6 Coherence KY.7.RP.2→ KY.8.F.4→ KY.HS.F.3
KY.8.F.5 Use graphs to represent functions. a. Describe qualitatively the functional relationship between two quantities by analyzing a graph. b. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **MP.3**, **MP.7**	Students describe whether a function is increasing or decreasing and linear or nonlinear. Function examples are described in contexts as well as in symbols. Coherence KY.7.RP.2→ KY.8.F.5→ KY.HS.F.4
Attending to the Standards for Mathematical Practice	
Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data (**MP.4**). There are many real-world problems that can be modeled with linear functions, including instances of constant payment plans (phone plans), costs associated with running a business and relationships between associated bivariate data. When students are analyzing graphs, they focus on how the function is changing. Students take verbal descriptions and create graphs, while also being able to take a graph and create a verbal description (**MP.2**, **MP.5**). Students look for patterns within the graphs to provide justification of the verbal description being represented by the graph (**MP.7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand congruence and similarity using physical models**, **transparencies**, **or geometry software.**	

Standards	Clarifications
KY.8.G.1 Verify experimentally the properties of rotations, reflections and translations: ● Lines are congruent to lines. ● Line segments are congruent to line segments of the same length. ● Angles are congruent to angles of the same measure. ● Parallel lines are congruent to parallel lines. **MP.5**, **MP.6**	Emphasis is congruence transformations preserve corresponding congruent lines, segments and angles. KY.HS.G.2 Coherence KY.8.G.1→ KY.HS.G.3(+)
KY.8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them. **MP.2**, **MP.7**	Students understand a figure, called a pre-image, is congruent to another figure, called the image, if the second figure can be obtained by a sequence of congruence transformations performed on the first figure. Students describe the sequence of congruence transformations necessary to transform one figure to a congruent second figure. KY.HS.G.4 Coherence KY.8.G.2→ KY.HS.G.5
KY.8.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates. **MP.3**, **MP.5**, **MP.6**	Emphasis is on noticing patterns across examples, noting how the x and y values change for different kinds of transformations. Coherence KY.8.G.3→ KY.HS.G.9
KY.8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **MP.2**, **MP.5**, **MP.7**	If similar, non-congruent figures are given, students understand a dilation must have taken place in the sequence of transformations to obtain the image from the pre-image. KY.HS.G.2 Coherence KY.8.G.4→ KY.HS.G.10

KY.8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. **MP.3**	Students use technology or physical tools to explore triangles. They arrange three copies of the same triangle so that the sum of the three angles appears to form a line and give an argument in terms of transversals of why this is so. KY.HS.G.7 Coherence KY.7.G.5→ KY.8.G.5→ KY.HS.G.10

Attending to the Standards for Mathematical Practice
Students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles and properties of polygons like rectangles and parallelograms (**MP.3**). With the aid of physical models, transparencies and geometry software, students in grade eight gain an understanding of transformations and their relationship to congruence of shapes (**MP.5**, **MP.6**). Through experimentation, students verify the properties of rotations, reflections and translations, including discovering these transformations change the position of a geometric figure but not its shape or size (**MP.7**). Finally, students understand congruent shapes are precisely those that can be "mapped" one onto the other by using rotations, reflections, or translations (**MP.2**).

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply the Pythagorean Theorem.	
Standards	**Clarifications**
KY.8.G.6 Explain a proof of the Pythagorean Theorem and its converse. **MP.3**, **MP.7**	Students verify, using a model, the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students understand if the sum of the squares of the two smaller legs is equal to the square of the third leg, then the triangle is a right triangle. Coherence KY.7.G.6→ KY.8.G.6→KY.HS.G.11
KY.8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **MP.1**, **MP.2**, **MP.4**	Students apply the Pythagorean Theorem to mathematical real-world problems. For example, finding the width of a television given the length and diagonal distance (two-dimensional) and the distance from the top left rear corner of a prism to the bottom right front corner of the prism (three-dimensional). Coherence KY.8.G.7→KY.HS.G.12
KY.8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **MP.5**, **MP.6**	Students calculate distances on the coordinate plane between two non-vertical or non-horizontal points by applying the Pythagorean Theorem. Students calculate distances between two non-vertical or non-horizontal points not given on a coordinate plane by applying the Pythagorean Theorem to absolute horizontal and vertical distances the student calculates. KY.HS.G.19 Coherence KY.8.G.8→ KY.HS.G.21
Attending to the Standards for Mathematical Practice	
By explaining a proof of the Pythagorean Theorem and its converse, students are constructing and defending arguments as to why the relationship is true (**MP.3**). The structure inherent in the use of the Theorem is a set of guidelines students seek to apply when applying the Theorem to right triangle relationships (**MP.7**). Students make sense of the world around them by applying the Pythagorean Theorem in a variety of ways (**MP.1**). Investigation into Pythagorean Triples and the relationships among similar triangles with the same ratio of Pythagorean Triples	

allows students to reason about the relationships (**MP.2**). Extending knowledge of the Pythagorean Theorem to the coordinate plane gives students another tool to prove the relationship exists and to apply the relationship to quantitative tasks (**MP.5**). Attending to precision is inherent in the study of this cluster, as a discussion will inevitably occur involving leaving a solution in terms of a radical, or a rational approximation ($\sqrt{50}\ vs.\ 7.07106\ldots$)(**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

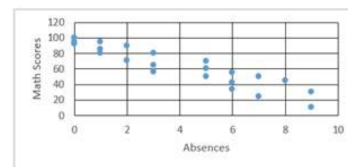
Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.**	
Standards	**Clarifications**
KY.8.G.9 Apply the formulas for the volumes and surface areas of cones, cylinders and spheres and use them to solve real-world and mathematical problems. **MP.1**, **MP.7**, **MP.8**	Cones: $V = \frac{1}{3}\pi r^2 h \quad SA = \pi r + (r + \sqrt{(r^2 + h^2)})$ Cylinders: $V = \pi r^2 h \quad SA = 2\pi rh + 2\pi r^2$ Spheres: $V = \frac{4}{3}\pi r^3 \quad SA = 4\pi r^2$ KY.HS.G.29 Coherence KY.7.G.4→ KY.8.G.9→ KY.HS.G.25
Attending to the Standards for Mathematical Practice	
Students may confuse the three formulas given if they try to apply a formula to a specific shape. Student understanding of the volume formulas is enhanced by investigations into the derivations of the volume formulas (**MP.1**). Students examining structure in real-world problems in order to apply the correct volume formula (if needed) begin to see where these are useful in real life (**MP.7**). If students can successfully compare volumes of similar shapes, for example, which of two storage tank can hold the most fuel, they begin to use repeated reasoning in the real-world (**MP.8**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

<table>
<tr><th colspan="2">Statistics and Probability</th></tr>
<tr><th colspan="2">Standards for Mathematical Practice</th></tr>
<tr><td>MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.</td><td>MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Cluster: Investigate patterns of association in bivariate data.</td></tr>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.8.SP.1 Construct and interpret scatter plots for bivariate numerical data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
MP.2, MP.7</td><td>

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80

Given data from students' math scores and absences, make a scatterplot.



For example, given the data and scatter plot to the left, students explain the relationship between students' absences and math scores shows a negative, linear association and has no obvious outliers.
KY.HS.SP.6
Coherence KY.8.SP.1→ KY.HS.SP.8</td></tr>
<tr><td>KY.8.SP.2 Know that lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a line and informally assess the model fit by judging the closeness of the data points to the line.
MP.2</td><td>Students are informally fitting a line to data; they judge whether or not a given line is a good fit for the data and describe needed adjustments. Students recognize some scatter plots cannot be described by a line.
KY.HS.SP.6
Coherence KY.8.SP.2→ KY.HS.SP.8</td></tr>
<tr><td>KY.8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate numerical data, interpreting the slope and intercept.
MP.2, MP.4</td><td>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height and an initial value of 4 cm means the plant was 4 cm tall when measuring began.
KY.HS.SP.6
Coherence KY.8.SP.3→ KY.HS.SP.7</td></tr>
</table>

Attending to the Standards for Mathematical Practice
Students reason quantitatively by symbolically representing the verbal description of a relationship between two bivariate variables. They attend to the meaning of data based on the context of problems and the possible linear or nonlinear functions that explain the relationships of the variables. When classifying characteristics of sets of data, students reason about the descriptions that apply based on definition (**MP.2**). Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data. This requires a deep understanding of the parts of the model used and their interpretations (**MP.4**). Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. Students identify patterns or structures in scatter plots. They fit lines to data displayed in a scatter plot and determine the equations of lines based on points or the slope and initial value (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1
Common Addition and Subtraction Situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	Group Size Unknown	Number of Groups Unknown
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3
Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\, x\, (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c then a > c
If a > b, then b < a
If a > b, then –a < -b
If a > b, then a ± c > b ± c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a ÷ c > b ÷ c
If a > b and c < 0, then a ÷ c < b ÷ c

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★