Kentucky Academic Standards
Mathematics

INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states and other parts of the world

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
    - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;
    - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
    - Become self-sufficient individuals;

- Become responsible members of a family, work group or community as well as an effective participant in community service;
- Think and solve problems in school situations and in a variety of situations they will encounter in life;
- Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
- Express their creative talents and interests in visual arts, music, dance, and dramatic arts.

● Increase student attendance rates.
● Reduce dropout and retention rates.
● Reduce physical and mental health barriers to learning.
● Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> *(Model Curriculum Framework,* page 19*)*

Legal Basis

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

704 KAR 8:040 Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;
- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and

- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.

- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).

- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)

- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

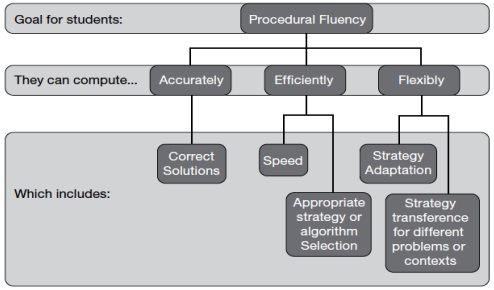
Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.

Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.



- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

<u>The Modeling Process</u>

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

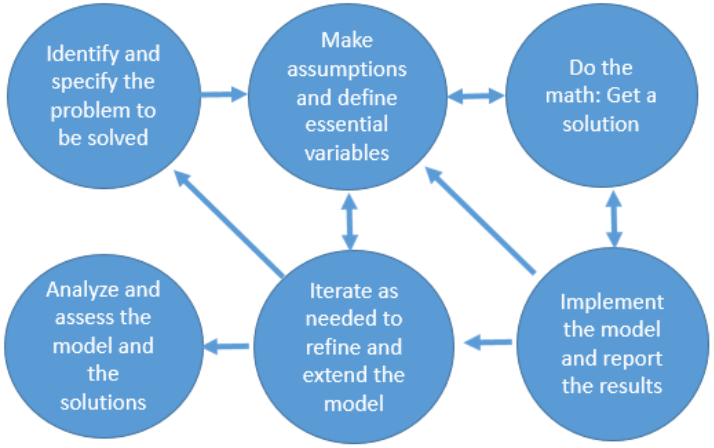
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

The Kentucky Academic Standards (KAS) are Standards, not Curriculum

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate "future-oriented" thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

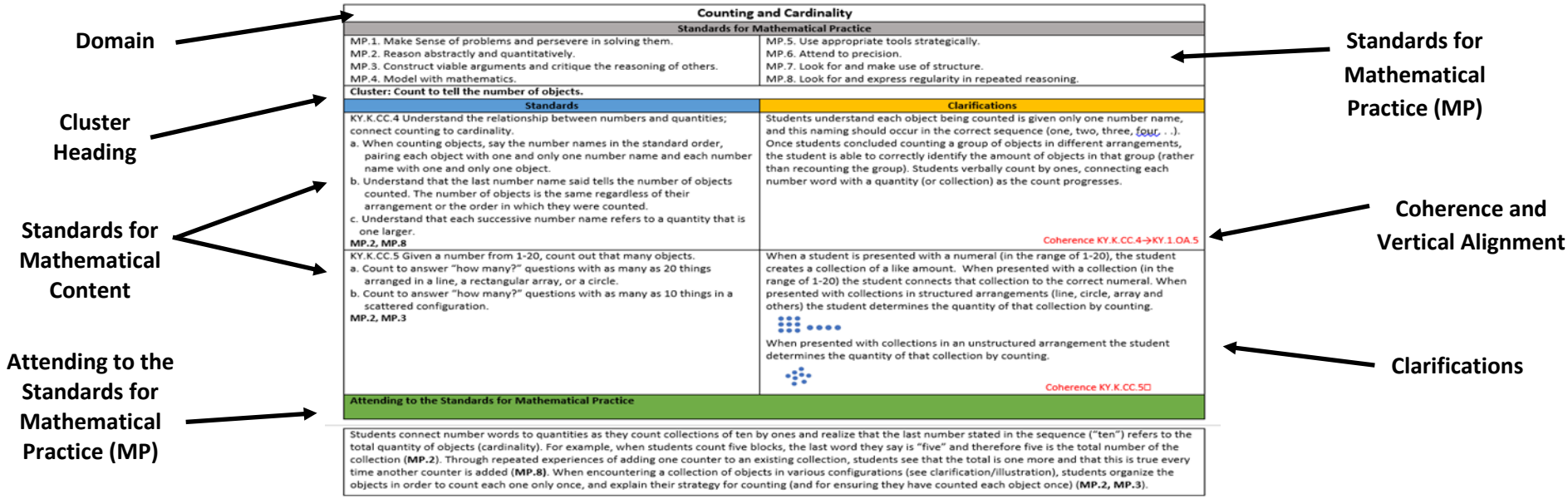
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

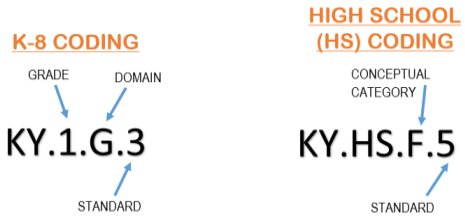
Standards for Mathematical Content define what students should understand and be able to do.

Standards for Mathematical Practice define how students engage in mathematical thinking.

The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.



How to Read the Coding of the Standards



Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council's 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order

to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making

assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Supplementary Materials to the Standards

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

Appendix A: Tables

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

Appendix B: Writing and Review Teams

Kentucky Academic Standards for Mathematics: Conceptual Category Number and Quantity

Number and Quantity Overview

The Real Number System	Quantities	The Complex Number System	Vector and Matrix Quantities
• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.	• Reason quantitatively and use units to solve problems.	• Perform arithmetic operations with complex numbers. • Represent complex numbers and their operations on the complex plane. • Use complex numbers in polynomial identities and equations.	• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Number and Quantity-The Real Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Extend the properties of exponents to rational exponents.	

Standards	Clarifications
KY.HS.N.1 Extend the properties of integer exponents to rational exponents, allowing for the expression of radicals in terms of rational exponents. **MP.2**, **MP.7**	Students understand that a single root can be expressed as a rational exponent with a numerator of one and a base that is equal to the root index. Students understand that powers and roots can be concisely expressed as a single rational exponent where the numerator is the power and the denominator is the root index. For example, students understand that defining $4^{1/3}$ is the same as the cube root of 4 because $4^{(1/3)^3} = (4^{1/3})^3$ so $4^{(1/3)^3}$ must equal 4.
KY.HS.N.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. **MP.7**	Standards KY.HS.N.2 builds on standard KY.HS.N.1 by extending student understanding to situations where the numerator is not one. For example, students understand that defining $4^{m/n}$ is the same as $\sqrt[n]{4}^{m}$ and $(\sqrt[n]{4})^{m}$. Include contextual examples, such as rewriting the volume of a sphere to identify the radius as a function of volume.

Attending to the Standards for Mathematical Practice
Students flexibly move between notating expressions as roots/powers or as integers with rational coefficients (**MP.2**). They explain why rational expressions can be more desirable and what the notation means (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Real Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use properties of rational and irrational numbers.	

Standards	Clarifications
KY.HS.N.3 (+) Justify why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **MP.3**, **MP.6**	An important difference between rational and irrational numbers is that rational numbers form a number system. Students understand that if you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Students also understand that multiplying the irrational number $\sqrt{2}$ by itself, yields a rational number, 2. Irrational numbers are defined by not being rational and this definition can be exploited to generate many examples of irrational numbers from just a few.

Attending to the Standards for Mathematical Practice
Students say or write what makes a number rational or irrational and use these definitions precisely to explain the properties of rational and irrational numbers **(MP.6)**. As students listen to the rationales or proofs of their peers, they determine whether the arguments make sense and prove the properties for all rational and irrational numbers **(MP.3)**.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason quantitatively and use units to solve problems.	

Standards	Clarifications
KY.HS.N.4 Use units in context as a way to understand problems and to guide the solution of multi-step problems; ★ a. Choose and interpret units consistently in formulas; b. Choose and interpret the scale and the origin in graphs and data displays. **MP.5**, **MP.6**	Graphical representations and data displays include but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots and multi-bar graphs.
KY.HS.N.5 Define appropriate units in context for the purpose of descriptive modeling. ★ **MP.1**, **MP.6**	In real-world situations, answers are usually represented by numbers with units. Units involve measurement, which requires precision and accuracy. For example, students should recognize that units measuring speed would not be appropriate for situations involving volume. Additionally students should understand when one dimensional, two dimensional, or three dimensional units are most applicable.
KY.HS.N.6 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★ **MP.2**, **MP.6**	While KY.HS.N.6 does not require a formal discussion or use of significant digits in the scientific sense, students understand a level of precision. For example, when using the Pythagorean Theorem with measurements given in tenths of an inch, it is appropriate for students to express answers to the nearest tenth, but not to the nearest hundredth because that level of precision was not used in the original measures.

Attending to the Standards for Mathematical Practice

Students attend to units in real-world problems, reasoning about the level of precision needed and the related error that may be introduced to the problem **(MP.2)**. Students describe what is (and is not) an appropriate level of precision for their answers, describing the relationship between the precision that was used in the original measures and the precision that can be used in an answer (**MP.6**).

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform arithmetic operations with complex numbers.	

Standards	Clarifications
KY.HS.N.7 Understanding properties of complex numbers. a. Know there is a complex number *i* such that $i^2 = -1$ and every complex number has the form $a + bi$ with *a* and *b* real. b. Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. **MP.7**, **MP.8**	a. Students understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations. c. Students understand the complex conjugate as the pair of binomial complex factors, (a + b*i*) (a – b*i*), whose product is a difference of squares: a^2+b^2, which is a real number. Students understand that the denominator of a fraction can be resolved of an imaginary number by multiplying by both the numerator and the denominator by the conjugate of the denominator.

Attending to the Standards for Mathematical Practice

Students use the relation $i^2 = -1$ as a basis for describing properties and then apply those properties to solving problems (**MP.7**). As they solve sets of problems with complex numbers, they notice patterns. For example, students explain how multiplying complex numbers is both alike and different from multiplying binomial expressions (**MP.8**).

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent complex numbers and their operations on the complex plane.	

Standards	Clarifications
KY.HS.N.8 (+) Understanding representations of complex numbers using the complex plane. a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. b. Represent addition, subtraction, multiplication, modulus and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. **MP.2**, **MP.5**	a. Students graph in both rectangular and polar form and convert rectangular coordinates to polar coordinates and vice versa. Students understand this conversion preserves the equality of the two forms. c. Students understand that calculating the distance between numbers in the complex plane is fundamentally the same as calculating distances in the standard coordinate plane using the distance formula from grade 8. Students understand calculating the midpoint of a segment in the complex plane as the average of the *a* values and average of the *b* values in any two endpoints expressed as *a +bi.*

Attending to the Standards for Mathematical Practice
Students use technology to graph complex numbers in rectangular and polar form (**MP.5**) and explain how these representations are equivalent (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use complex numbers in polynomial identities and equations.	

Standards	Clarifications
KY.HS.N.9 Solve quadratic equations with real coefficients that have complex solutions. **MP.1**, **MP.2**	Students use the Quadratic Formula to solve for complex solutions. Students recognize that when a quadratic equation yields complex solutions its graph does not cross the x-axis.
KY.HS.N.10 (+) Extend polynomial identities to the complex numbers. **MP.7**, **MP.8**	When multiplying complex binomials, students recognize and understand the value of i^2 as -1 and fluently simplify each polynomial appropriately navigating between the real number system and complex numbers. One example of this might be that students should understand that it would be appropriate to rewrite x^2+4 as $(x+2i)(x-2i)$.
KY.HS.N.11 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **MP.1**, **MP.3**	

Attending to the Standards for Mathematical Practice
Students make sense of quadratic equations, looking to see if there are rational roots that can be found by factoring, or if other methods such as completing the square or the quadratic formula are needed (**MP.1**). They justify that their answer is reasonable and describe why there are no real roots, if that is the case (**MP.2**).

Number and Quantity-Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and model with vector quantities.	

Standards	Clarifications
KY.HS.N.12 (+) Understand and apply properties of vectors. a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. c. Solve problems involving velocity and other quantities that can be represented by vectors. **MP.1**, **MP.6**	a. Vectors are directed by an angle and continue in that direction for a set length. b. Students connect 1) finding vertical and horizontal components and the magnitude of a vector with 2) using the Pythagorean Theorem in the coordinate plane. Limit to two-dimensional vectors.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity- Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform operations on vectors.	

Standards	Clarifications
KY.HS.N.13 (+) Perform operations with vectors (addition, subtraction and multiplication by a scalar). a. Add vectors end-to-end, component-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise. d. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. e. b. Compute the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). **MP.3**, **MP.7**	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform operations on matrices and use matrices in applications.	

Standards	Clarifications
KY.HS.N.14 Use matrices to represent and manipulate data. **MP.4**, **MP.5**	Students understand matrices are rectangular arrays comprised of elements that are useful for solving problems in context.
KY.HS.N.15 Perform operations with matrices. a. Add, subtract and multiply matrices of appropriate dimensions. b. Multiply matrices by scalars to produce new matrices. **MP.7**, **MP.8**	
KY.HS.N.16 (+) Understand properties of square and identity matrices. a. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. b. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. c. Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. **MP.3**, **MP.7**	
KY.HS.N.17 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **MP.2**, **MP.7**	

Attending to the Standards for Mathematical Practice
Students create numerical arrays of data (matrices), taken from a variety of sources (e.g., tables, systems of equations, or coordinate points from a series of transformations) (**MP.2**) **and** they use technology to manipulate data when appropriate (**MP.5**). When performing matrix operations by hand, students look for patterns and make generalizations (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Conceptual Category Algebra

Algebra Overview

Seeing Structure in Expressions	Arithmetic with Polynomials and Rational Expressions	Creating Equations ★	Reasoning with Equations and Inequalities
• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems.	• Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions.	• Create equations that describe numbers or relationships.	• Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations. • Represent and solve equations and inequalities graphically.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Algebra-Seeing Structure in Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret the structure of expressions.	

Standards	Clarifications
KY.HS.A.1 interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors and coefficients. b. Interpret complicated expressions, given a context, by viewing one or more of their parts as a single entity. **MP.2**, **MP.6**	Students encounter simpler scenarios where they interpret r·t as the product of a given rate and time or interpret the perimeter expression (2*l*+2*w*) contextually as the sum of twice the length and twice the width of a rectangle. Students encounter more complicated scenarios where they interpret $P(1+r)^n$ contextually as the product of a principal investment, P and $(1+r)^n$ which represents an investment rate, compounding factor and time.
KY.HS.A.2 Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. **MP.7**, **MP.8**	Students see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares factored as $(x^2 – y^2)(x^2 + y^2)$. Additionally, students see there are three commonly used forms for a quadratic expression: • Standard form • Factored form • Vertex form and can identify when one form might be more useful than another.

Attending to the Standards for Mathematical Practice

Students not only simplify problems, they use vocabulary, such as terms, coefficients and degrees, appropriately as they describe their process (**MP.6**). Students describe the meaning of parts of an expression, such as a particular term or coefficient and also explain the meaning of the full expression (**MP.7**). Students fluently manipulate expressions into equivalent forms, based on patterns they have noticed across problems (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Seeing Structure in Expressions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Write expressions in equivalent forms to solve problems.

Standards	Clarifications
KY.HS.A.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient and constant term. b. Factor a quadratic expression to reveal the zeros of the function it defines. c. Use the properties of exponents to rewrite exponential expressions. d. (+) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **MP.5**, **MP.7**	KY.HS.A.3b Students recognize the connection between the zero product property and solving a quadratic in one variable by setting factored expressions equal to zero. KY.HS.A.3c KY.HS.A.3d (+) Students recognize being able to complete the square allows them to identify the coordinates of the maximum or minimum value more easily than when the quadratic is in standard form and there are pros and cons of each equivalent form.
KY.HS.A.4 (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. ★ **MP.1**, **MP. 4**	$s_n = \frac{a_1 - a_1 r^n}{1-r}$ where r ≠1

Name	Product of Powers	Quotient of Powers	Power of a Product	Power of a Quotient	Power of a Power	Negative Exponent
Property	$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a \cdot b)^n = a^n \cdot b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(a^m)^n = a^{mn}$	$a^{-n} = \frac{1}{a^n}$

Attending to the Standards for Mathematical Practice

Students explain that they need to rewrite quadratic expressions into equivalent factored forms in order to find the zeros of the function it defines (**MP.7**). Using technology, students change the exponents to reinforce their understanding of exponent properties (**MP.5**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform arithmetic operations on polynomials.	

Standards	Clarifications
KY.HS.A.5 Add, subtract and multiply polynomials. **MP.7**, **MP.8**	Students combine like terms and make use of the distributive property when adding, subtracting and multiplying polynomials.

Attending to the Standards for Mathematical Practice
Students flexibly rewrite expressions in equivalent forms using algebraic properties, including properties of addition, subtraction and multiplication (**MP.7**). When multiplying binomials, students identify and describe shortcuts after noticing that calculations are repeated (**MP.8**).

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the relationship between zeros and factors of polynomials.	

Standards	Clarifications
KY.HS.A.6 (+) Know and apply the Remainder Theorem. **MP.1**, **MP.8**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
KY.HS.A.7 Identify roots of polynomials when suitable factorizations are available. Know these roots become the zeros (x-intercepts) for the corresponding polynomial function. **MP.2**, **MP.5, MP.7**	Methods of finding roots could include, but are not limited to: • factoring • synthetic division • long division • an analysis of the graph (created by hand or through use of technology).

Attending to the Standards for Mathematical Practice
Students reason quantitatively as they select a method for finding roots and justify why they selected and applied a particular method (**MP. 2**). Students use technology to identify the *x*-intercepts from a polynomial graph and explain that the *x*-intercepts are zeros and therefore roots of the polynomials (**MP.5**).

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use polynomial identities to solve problems.	

Standards	Clarifications
KY.HS.A.8 (+) Prove polynomial identities and use them to describe numerical relationships. **MP.2**, **MP.3**, **MP.6**	Students observe the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
KY.HS.A.9 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. **MP.7**, **MP.8**	Students understand the Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Rewrite rational expressions.	

Standards	Clarifications
KY.HS.A.10 (+) Rewrite simple rational expressions in different forms. **MP.7**, **MP.8**	Students observe how to write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$. Methods of rewriting rational expressions could include, but are not limited to: • Inspection • Synthetic division • Long division • Use of technology
KY.HS.A.11 (+) Add, subtract, multiply and divide rational algebraic expressions. **MP.2**, **MP.3**	Students go beyond demonstrating procedural fluency and apply this standard in a variety of contextual situations.

Algebra-Creating Equations ★	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Create equations that describe numbers or relationships.	

Standards	Clarifications
KY.HS.A.12 Create equations and inequalities in one variable and use them to solve problems. **MP.1**, **MP.4**	Students use the addition, subtraction, multiplication and division properties for both equations and inequalities to solve problems. These equations may arise from linear and quadratic functions and simple rational and exponential functions.
KY.HS.A.13 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **MP.2**, **MP.5**	Students solve systems of equations with two or more variables to solve problems in the real world setting.
KY.HS.A.14 Create a system of equations or inequalities to represent constraints within a modeling context. Interpret the solution(s) to the corresponding system as viable or nonviable options within the context. **MP.4**, **MP.5**	Students may be asked to find an optimal solution and the conditions under which the optimal solution would occur for a given real world situation.
KY.HS.A.15 Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations. **MP.2**, **MP.7**	Students encounter scenarios where they rewrite formulas/equations for variables different from the commonly used formulas. An example may include, but not being limited to, students rearranging Ohm's law (V = IR) to highlight resistance R, rather than the variable for voltage V.

Attending to the Standards for Mathematical Practice
Students interpret a story or situation into an equation or inequality, connecting the terms and symbols within the equation or inequality to the context (**MP.1**) and relate how the solution to the equation or inequality connects back to the original problem (**MP.4**). Students utilize technology to graph equations and use the graph to describe qualitatively and quantitatively the relationship between variables (**MP.5**). Students explain when they would opt for different equivalent forms an equation (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand solving equations as a process of reasoning and explain the reasoning.	
Standards	**Clarifications**
KY.HS.A.16 Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **MP.1**, **MP.3**	Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x+ 4 = 10$, $2(x- 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc. After solving many linear equations in one variable, students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b= cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.
KY.HS.A.17 Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. a. Solve rational equations written as proportions in one variable. b. Solve radical equations in one variable. **MP.3**, **MP.5, MP.7**	Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation.
Attending to the Standards for Mathematical Practice	
Students use properties, such as the distributive property of multiplication over addition, to describe why two expressions are equivalent. They explain their approach to a problem, as well as critique the solutions of others, comparing the different approaches in terms of whether they are	

Attending to the Standards for Mathematical Practice
accurate and efficient (**MP.3**)**.** Students approximate solutions with technology (**MP.5**)**.** Students use structure of an equation (rational, radical), to determine an efficient strategy for finding a solution, if one exists (**MP.7**)**.**

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve equations and inequalities in one variable.	

Standards	Clarifications
KY.HS.A.18 Solve linear equations and inequalities in one variable, including literal equations with coefficients represented by letters. **MP.2**, **MP.7**	Students use all properties of both equations and inequalities to solve for one variable.
KY.HS.A.19 Solve quadratic equations in one variable. a. Solve quadratic equations by taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. b. (+) Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. c. (+) Solve quadratic equations by completing the square. **MP.1**, **MP.8**	Students observe that methods for solving quadratic equations are interrelated and certain situations may more appropriately call upon one method as opposed to the other methods. b & c. (+) Students understand completing the square involves factoring and the quadratic formula is nothing more than an encapsulation of the method of completing the square. While all students are not required to be able to use completing the square as a method for solving quadratic equations, exposure to this method is needed to explain how the quadratic formula is derived.

Attending to the Standards for Mathematical Practice
Students reason about which symbolic representation is needed in order to focus on a particular feature and then efficiently rewrite literal equations to feature that characteristic (**MP.2**)**.** Students analyze the structure of a quadratic equation to determine an efficient strategy to find a solution (**MP.7**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve systems of equations.	

Standards	Clarifications
KY.HS.A.20 Solve systems of linear equations in two variables. a. Understand a system of two equations in two variables has the same solution as a new system formed by replacing one of the original equations with an equivalent equation. b. Solve systems of linear equations with graphs, substitution and elimination, focusing on pairs of linear equations in two variables. **MP.3**, **MP.6**	a. This part of the standard is not focused on the actual process of solving a system of equations, but rather the proof of the method (specifically the elimination method). b. Students utilize a variety of methods to solve system of equations including graphing the system, solving using the substitution method, solving the system with elimination both with and without involving multiplication. Students recognize the conclusion of these processes may result in obtaining one solution (expressed as an ordered pair), no solution or infinitely many solutions.
KY.HS.A.21 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **MP.3**, **MP.6**	Students utilize algebra techniques and graphical representations to determine points of intersection between lines and parabolas that indicate solution sets for a system of linear and quadratic equations.
KY.HS.A.22 (+) Use matrices to solve a system of equations. a. Represent a system of linear equations as a single matrix equation in a vector variable. b. Find the inverse of a matrix if it exists. c. Use matrices to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **MP.4**, **MP.7**	a. Students do not focus on the solving of the system, but rather translating between the two different representations for this part of the standard. b. Methods of solving systems with matrices could include, but are not limited to: • utilizing inverse matrices • row reduction • Cramer's rule

Attending to the Standards for Mathematical Practice

Students use a variety of methods to solve systems of equations, understanding that tables and graphs may produce estimates rather than exact solutions (**MP.6**)**.** Students construct a viable argument to justify their solution(s) in a system of equations. (**MP.3**)

Algebra- Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and solve equations and inequalities graphically.	

Standards	Clarifications
KY.HS.A.23 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. **MP.1**, **MP.4**	Students make connections between algebra and geometry within this standard. Students acquire the basic understanding that the coordinates of the points of intersection of the graphs are the pairs of values of the variables that solve the system.
KY.HS.A.24 Justify that the solutions of the equations $f(x) = g(x)$ are the x-coordinates of the points where the graphs of $y = f(x)$ and $y = g(x)$ intersect. Find the approximate solutions graphically, using technology or tables. ★ **MP.3**, **MP.5**	Students justify solutions for equations which Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. ★
KY.HS.A.25 Graph linear inequalities in two variables. a. Graph the solutions to a linear inequality as a half-plane (excluding the boundary in the case of a strict inequality). b. Graph the solution set to a system of linear inequalities as the intersection of the corresponding half-planes. **MP.5**, **MP.6**	Students recall skills regarding graphing the solutions of a linear inequality in the coordinate plane in order to graph the solution set for a system of linear inequalities. Students utilize these skills in other standards via linear programming.

Attending to the Standards for Mathematical Practice
Students explain that the solutions of a system of equations or inequalities are all the points represented on the graph and therefore, where two functions overlap illustrates solutions to two functions (**MP.1, MP.3**). Students use technology to determine solutions to a system of linear inequalities (e.g., using DESMOS or graphing calculators) (**MP.5**)**.**

Kentucky Academic Standards for Mathematics: Conceptual Category Functions

Functions Overview

Interpreting Functions	Building Functions	Linear, Quadratic and Exponential Models	Trigonometric Functions
• Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of the context. • Analyze functions using different representations.	• Build a function that models a relationship between two quantities. • Build new functions from existing functions.	• Construct and compare linear, quadratic and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the concept of a function and use function notation.	

Standards	Clarifications
KY.HS.F.1 Understand properties and key features of functions and the different ways functions can be represented. a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. b. Using appropriate function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. c. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. d. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. e. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **MP.2**, **MP.4**, **MP.7**	a. When describing relationships between quantities, the defining characteristic of a function is the input value determines the output value or, equivalently, the output value depends upon the input value. In some situations where two quantities are related, each can be viewed as a function of the other. c. A function is often described and understood in terms of the output behavior, or over what input values is it increasing, decreasing, or constant. Important questions include, "For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?" Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions. Key features include, but are not limited to: intercepts; intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior; periodicity. e. Students compare characteristics from various representations for one type of family of function at a time. For quadratics, students might determine which function has the larger maximum when given two different representations of quadratic functions.

KY.HS.F.2 Recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **MP.7**, **MP.8**	Sequences are functions with a domain consisting of whole numbers.

Attending to the Standards for Mathematical Practice
Students reason quantitatively about the relationship between domain and range of functions across abstract and concrete representations (**MP.2**). Students look closely to discern arithmetic and geometric relationships as patterns with additive and multiplicative changes, respectively **(MP.7)**. Students notice the regularity in the pattern to write a general formula for arithmetic or geometric sequence (**MP.8**).

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret functions that arise in applications in terms of the context.	

Standards	Clarifications
KY.HS.F.3 Understand average rate of change of a function over an interval. a. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. b. Estimate the rate of change from a graph. ★ **MP.2**, **MP.4**	The rate of change over an interval is equivalent to the slope between the endpoints of the interval. For linear functions, the rate of change is constant, over all intervals. However, for nonlinear functions, the average rate of change may vary depending on the interval.

Attending to the Standards for Mathematical Practice
Students make sense of the rate of change, recognizing it captures how the input and the output of a function vary simultaneously (**MP. 2**). For example, students explain that the rate of change for nonlinear functions is not constant. Students use equations, tables and graphs to analyze rate of change in applied and mathematical contexts (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Analyze functions using different representations.	

Standards	Clarifications
KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★ a. Graph linear and quadratic functions and show intercepts, maxima and minima. b. Graph square root, cube root and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior. e. (+) Graph trigonometric functions, showing period, midline and amplitude. f. (+) Graph piecewise functions, including step functions. g. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. **MP.4**, **MP.5**	Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features important for identifying and describing functions. This standard indicates the function families in students' repertoires, detailing which features are required for several key families. Students demonstrate fluency with linear, quadratic and exponential functions, including the ability to graph without using technology. In other function families, students graph simple cases without technology and more complex ones with technology.
KY.HS.F.5 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Identify zeros, extreme values and symmetry of the graph within the context of a quadratic function.	a. Quadratic functions provide a rich playground for developing this ability, since the three principal forms for a quadratic expression (expanded, factored and completed square) each give insight into different aspects of the function. b. Students examine real-world situations with constant multiplicative change, represented as expressions, such as growth or decay.

b. Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3**, **MP.6**	

Attending to the Standards for Mathematical Practice
Students use graphs to answer questions and/or make predictions for a given context (**MP. 4**). Students use technology to explore concepts of function families and show key features of the graph (**MP. 5**). Students compare and contrast different characteristics of functions to connect features of the graph with different real-world contexts (**MP.6**). Students manipulate expressions, being careful to preserve equivalence and describe why a particular expression provides insights into the function (**MP.3, MP.6**).

Functions-Building Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Build a function that models a relationship between two quantities.	
Standards	**Clarifications**
KY.HS.F.6 Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions. **MP.4**, **MP.7**	b. Use real-world examples when appropriate. c. Consider contextual examples for composition functions, such as, if $T(y)$ is the temperature in the atmosphere as a function of height and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
KY.HS.F.7 Use arithmetic and geometric sequences to model situations and scenarios. a. Use formulas (explicit and recursive) to generate terms for arithmetic and geometric sequences. b. Write formulas to model arithmetic and geometric sequences and apply those formulas in realistic situations. ★ c. (+) Translate between recursive and explicit formulas. **MP.4**, **MP.8**	Examples include, but are not limited to: • calculating mortgages • drug dosages • simple interest
Attending to the Standards for Mathematical Practice	
For real-world problems, students formulate the problem, make assumptions, define variables and create functions to model the situation (**MP.4**)**.** Students notice the regularity in real-world growing patterns and use these insights to write a general formula to describe arithmetic or geometric sequences (**MP.8**).	

Functions- Building Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Build new functions from existing functions.	

Standards	Clarifications
KY.HS.F.8 Understand the effects of transformations on the graph of a function. a. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, k $f(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. b. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **MP.3**, **MP.5**	a. Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.
KY.HS.F.9 Find inverse functions. a. Given the equation of an invertible function, find the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2**, **MP.6**	a. Students can complete the process of finding the inverse when given an equation of a function that is invertible. b-d. Students need a formal sense of inverse functions. Students understand a function and its inverse describe the exact same relationship but in different ways.
KY.HS.F.10 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents with the use of technology. **MP.1**, **MP.7**	Students can use inverses of simple logarithmic and exponential equations in order to solve those equations. The inverse relationship between logarithmic and exponential functions is special in that each function's inverse is also a function

Attending to the Standards for Mathematical Practice
Students use technology to explore how changing the value of k impacts the graph of the function (**MP.5**)**.** Students use the graphical representation to create plausible arguments about the effects of transformations, instead of relying on computational rules (**MP.3**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Construct and compare linear, quadratic and exponential models and solve problems.	

Standards	Clarifications
KY.HS.F.11 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Recognize and justify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **MP.3**, **MP.8**	Linear functions have the same average rate of change over same-sized intervals; the same value is added to the output over each interval. In contrast, the outputs of exponential functions grow or decay by the same percent over same-sized intervals; the same value is multiplied by the output over each interval.
KY.HS.F.12 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **MP.7**, **MP.8**	Students construct functions with and without technology.
KY.HS.F.13 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7**, **MP.8**	Students compare functions by focusing on how the output values change over intervals of equal length. Even though a linear function may initially be increasing faster than an exponential function, an increasing exponential function always eventually exceeds an increasing linear function.

Attending to the Standards for Mathematical Practice
Students reason about particular characteristics of linear, quadratic and exponential functions, for example comparing how rates of change across different types of functions (**MP.3**). Students recognize families of functions in a more general sense to discern that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically (**MP.8**).

Functions- Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret expressions for functions in terms of the situation they model.	

Standards	Clarifications
KY.HS.F.14 Interpret the parameters in a linear or exponential function in terms of a context. **MP.1**, **MP.2**	More than just substituting values into a given formula, this requires students to understand how changing specific parameters will change the function output. An example of this with an exponential function ($f(x) = a \bullet b^x$) might be changing the "b" from a number greater than 1 to a number between 0 and 1. Students should recognize this creates a decay problem instead of a growth problem. Similarly, changing the "a" parameter creates corresponding changes to the graph and has different implications within the realistic context.

Attending to the Standards for Mathematical Practice
Students quantitatively reason to consider the units, limitations and parameters in linear and exponential functions in terms of a context (**MP.2**). When solving problems, students ask themselves, "Does this make sense?" (**MP.1**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Extend the domain of trigonometric functions using the unit circle.	

Standards	Clarifications
KY.HS.F.15 (+) Understand the relationship of radian measure of an angle to its arc length. **MP.1**, **MP.6**	Understanding radian measure of an angle as arc length on the unit circle enables students to build on their understanding of trigonometric ratios associated with acute angles and to explain how these ratios extend to trigonometric functions whose domains are included in the real numbers.
KY.HS.F.16 (+) Understand and use the unit circle. a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. b. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$ and use the unit circle to express the values of sine, cosine and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x, where x is any real number. c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MP.7**, **MP.8**	This standard is sometimes called "unwrapping the unit circle." For each function, the angle θ is represented by values on the horizontal axis and the resulting outputs are graphed on the vertical axis. c. Students understand symmetry exists within the unit circle for paired reference angles: $\sin(-\theta) = -\sin(\theta)$, so sine is an odd function; and $\cos(-\theta) = \cos(\theta)$, so cosine is an even function.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Model periodic phenomena with trigonometric functions.	

Standards	Clarifications
KY.HS.F.17 (+) Choose trigonometric functions to model periodic phenomena with specified period, midline and amplitude. ★ **MP.4**, **MP.5, MP.6**	A function is described as sinusoidal or is called a sinusoid if it has the same shape as the sine graph, for example, has the form $f(t) = A + B(\sin Ct + D)$. Many real-world phenomena can be approximated by sinusoids, including sound waves, oscillation on a spring, the motion of a pendulum, tides and phases of the moon. Because $\sin(t)$oscillates between 1 and 1, $A + B(\sin Ct + D)$will oscillate between $A - B$ and $A + B$. Thus, $y = A$ is the midline and B is the amplitude of the sinusoid. Students can obtain the frequency of f: the period of $\sin(t)$ is 2π, so (knowing the effect of multiplying t by C) the period of $\sin(Ct)$ is $2\pi/C$ and the frequency is its reciprocal. When modeling, students have the sense that C affects the frequency and that C and D together produce a phase shift, but finding a correct solution might involve technological support, except in simple cases.
KY.HS.F.18 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MP.2**, **MP.3**	Students experience restricting the domain of a function so it has an inverse. For trigonometric functions, a common approach to restricting the domain is to choose an interval on which the function is always increasing or always decreasing.

KY.HS.F.19 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. ★ **MP.4**, **MP.5**	Include $\sin^{-1} x, \cos^{-1} x \ and \ \tan^{-1} x$.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Prove and apply trigonometric identities.	
Standards	**Clarifications**
KY.HS.F.20 (+) Proving identities and formulas within the context of trigonometry. a. Prove the Pythagorean identity and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. b. Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems. **MP.3**, **MP.7**	In the unit circle, the x-value is the cosine and the y-value represents the sine. Since the hypotenuse of any right triangle on the unit circle is 1, the Pythagorean relationship of $x^2 + y^2 = 1$ holds. Students connect the Pythagorean Theorem in geometry and the study of trigonometry to understand this relationship.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Conceptual Category Geometry

Geometry Overview

Congruence	Similarity, Right Triangles and Trigonometry	Circles	Expressing Geometric Properties with Equations	Geometric Measurement and Dimensions	Modeling with Geometry
• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Prove geometric theorems. • Make geometric constructions.	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. • Define trigonometric ratios and solve problems involving right triangles. • Apply trigonometry to general triangles.	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles.	• Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically.	• Explain volume formulas and use them to solve problems. • Visualize relationships between two-dimensional and three-dimensional objects	• Apply geometric concepts in modeling situations.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

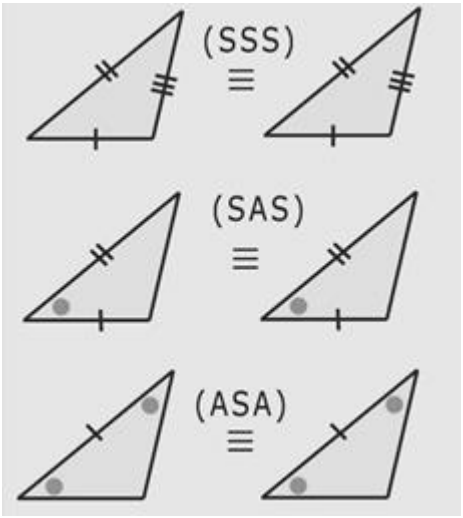
Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Experiment with transformations in the plane.	
Standards	**Clarifications**
KY.HS.G.1 Know and apply precise definitions of the language of Geometry: a. Understand properties of line segments, angles and circle. b. Understand properties of and differences between perpendicular and parallel lines. **MP.3**, **MP.6**	Students in high school start to formalize the intuitive geometric notions they developed in grades 6–8 and give specificity to geometric concepts that can serve as a good basis for developing precise definitions and arguments. a. Students understand a more formal knowledge of postulates, theorems and various properties relating to line segments, angles and circles. This knowledge is based on the undefined notions of point, line, distance along a line and distance around a circular arc. b. Students understand important properties of both parallel and perpendicular lines, prior to making the connections between these types of lines and how they relate to their calculated or given slope.
KY.HS.G.2 Representing transformations in the plane. a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs b. Compare transformations that preserve distance and angle measures to those that do not. c. Given a rectangle, parallelogram, trapezoid, or regular polygon, formally describe the rotations and reflections that carry it onto itself, using properties of these figures. **MP.5**, **MP.7**	Software, transparencies, etc. may be used to accurately represent congruence transformations in the plane. a. Students understand any point (a,b) can be thought of as an input and any image of point (a,b) can be thought of as the output of a specific transformation function. b. Students make connections between which transformations are a rigid motion (isometry) and which transformations do not have that characteristic. c. Students practice and understand the procedures needed to carry out multiple transformations that carry the figure onto itself, recognizing the important properties of these figures.

KY.HS.G.3 (+) Develop formal definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. **MP.6**, **MP.7**	Students understand and recognize characteristics of various transformations of multiple different geometric figures. Students develop formal definitions that reflect those transformations.
KY.HS.G.4 Understand the effects of transformations of geometric figures. a. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. b. Specify a sequence of transformations that will carry a given figure onto another. c. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **MP.2**, **MP.8**	Students understand a figure, called a pre-image, is congruent to another figure, called the image, if that second figure can be obtained by a sequence of congruence transformations performed on the first figure. Students can draw the image of a transformed pre-image using a variety of tools, including but not limited to: • graph paper • manipulatives • tracing paper • computer programs Students perform such sequences and describe the sequence of congruence transformations necessary to transform one figure to an congruent second figure.

Attending to the Standards for Mathematical Practice
Students make careful calculations when transforming figures by hand (**MP.6**) and use technology (**MP.5**) to analyze more complicated cases and to make generalizations (**MP.7**). Students use correct terminology when discussing figures and the effects of their transformed figure (**MP.3, MP.6**), identifying congruent, distance-preserving, figures when possible. For example, students connect geometric transformations with algebra when comparing a figure *F* and the transformed figure *T(F)* or a figure that has undergone multiple transformations *T(R(F))* (**MP.2**).

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand congruence in terms of rigid motions.	

Standards	Clarifications
KY.HS.G.5 Know and apply the concepts of triangle congruence: a. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Explain how the criteria for triangle congruence (ASA, SAS and SSS) follow from the definition of congruence in terms of rigid motions. **MP.3**, **MP.6**	

Attending to the Standards for Mathematical Practice
Students fluently and intentionally select and/or calculate measures (**MP.6**) when deliberating criteria for triangle congruence (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Prove geometric theorems.	

Standards	Clarifications
KY.HS.G.6 Apply theorems for lines, angles, triangles, parallelograms. **MP.2**, **MP.3**	Students use previously learned definitions, theorems, postulates and properties of lines, angles, triangles and parallelograms to draw conclusions and to make inferences. Theorems for lines and angles include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Theorems for triangles include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Theorems for parallelograms include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.
KY.HS.G.7 Prove theorems about geometric figures. a. Construct formal proofs to justify theorems for lines, angles and triangles. b. (+) Construct formal proofs to justify theorems for parallelograms. **MP.6**, **MP.7**	Students recall definitions, theorems, postulates and properties to construct formal proofs based on theorems established in other standards. (+)Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.

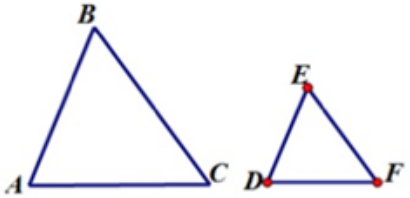
Attending to the Standards for Mathematical Practice
Students experiment with lines, angles, triangles and parallelograms to make connections and conjectures about their properties (**MP.7**), using dynamic software when appropriate (**MP.5**). Students routinely use various forms of proof (formal, informal, direct and indirect) to outline their logic and defend their conjectures (**MP.3**). Students consider alternate approaches to a proof or a conjecture and debate the alternatives for effectiveness and accuracy (**MP.2**, **MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Make geometric constructions.	
Standards	**Clarifications**
KY.HS.G.8 Create and apply geometric constructions. a. Make formal geometric constructions with a variety of tools and methods. b. Apply basic construction procedures to construct more complex figures. **MP.5**, **MP.6**	Methods for formal constructions may include but are not limited to: • compass and straightedge • string • reflective devices • paper folding • technology Students demonstrate the ability to copy a segment, copy an angle, bisect a segment, bisect an angle, construct perpendicular lines which includes the perpendicular bisector of a line segment and construct a line parallel to a given line through a point not on the line.
Attending to the Standards for Mathematical Practice	
Students select and use a variety of tools to generate geometric constructions (**MP.5**). Students use precision when constructing shapes and figures by hand and select and use appropriate technology for complicated constructions (**MP.5, MP.6**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

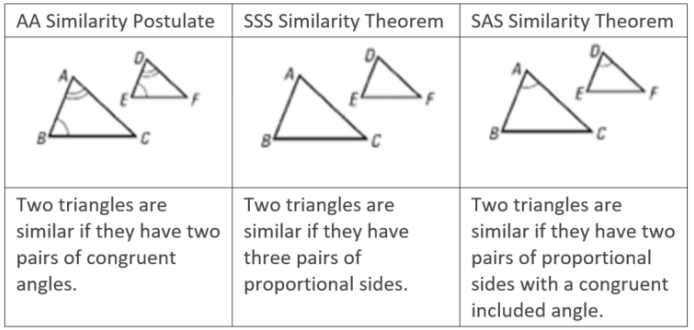
Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand similarity in terms of similarity transformations.	

Standards	Clarifications
KY.HS.G.9 Understand properties of dilations. a. Verify the properties that result from that dilations given by a center and a scale factor. b. Verify that a dilation produces an image that is similar to the pre-image. **MP.5**, **MP.7**	• Methods to verify properties could include, but not limited to: scale models, moving an object closer to a light source and examining changes, changing the scale factor on a copier. • Students explain the effect of dilations on objects that pass through the center verses those that do not pass through the center of a figure. • Students understand within this standard, the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides is a result that occurs because two objects are similar.
KY.HS.G.10 Apply the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **MP.3**, **MP.6**	The AA Similarity Theorem  If $\angle A \cong \angle D, and\ \angle B \cong \angle E,$ Then $\Delta ABC \sim \Delta DEF.$

Attending to the Standards for Mathematical Practice
With the aid of physical models, transparencies and geometry software, students verify whether figures are similar or not (**MP.5**, **MP.6**). As they compare similar shapes, they make generalizations about what changes and what stays the same when, and use this information to do dilations (**MP.7**). Students prepare illustrations and explanations related to the AA triangle similarity criterion, as well as by considering and discussing properties of similar triangles (**MP.3**).

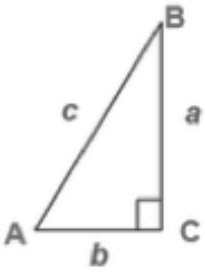
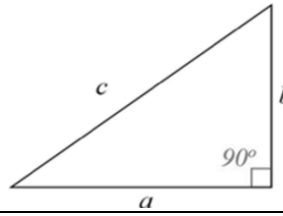
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Prove theorems involving similarity.	

Standards	Clarifications
KY.HS.G.11 Understand theorems about triangles. a. Apply theorems about triangles. b. (+) Prove theorems about triangles. c. Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **MP.1**, **MP.3**	Theorems include the Pythagorean Theorem and "a line parallel to one side of a triangle divides the other two proportionally and conversely." Students demonstrate the ability to copy a segment, copy an angle, bisect a segment, bisect an angle, construct perpendicular lines, which includes the perpendicular bisector of a line segment and construct a line parallel to a given line through a point not on the line. Triangle Similarity Postulate and Theorems:  AA Similarity Postulate: Two triangles are similar if they have two pairs of congruent angles. SSS Similarity Theorem: Two triangles are similar if they have three pairs of proportional sides. SAS Similarity Theorem: Two triangles are similar if they have two pairs of proportional sides with a congruent included angle.

Attending to the Standards for Mathematical Practice
Students identify cases where the AA triangle similarity criterion can be used (**MP.1**) and routinely use various methods of proof (formal, informal, direct and indirect) to outline their logic in order to defend their conjectures (**MP.3**).

Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Define trigonometric ratios and solve problems involving right triangles.	

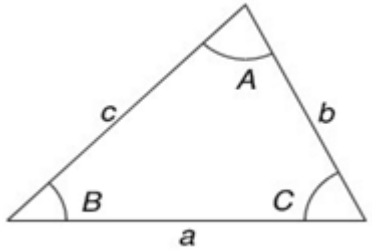
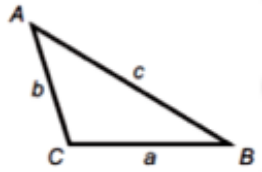
Standards	Clarifications
KY.HS.G.12 Understand properties of right triangles. a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles (sine, cosine and tangent). b. Explain and use the relationship between the sine and cosine of complementary angles. c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★ **MP.3**, **MP.4**	 $\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$ $\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$ $\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$  $c^2 = a^2 + b^2$

Attending to the Standards for Mathematical Practice

Given a variety of similar triangles, students compare ratios of corresponding pairs of sides in order to discover the definitions of trigonometric ratios for acute angles (**MP.3**). Students use these trigonometric ratio definitions to solve real-world problems involving right triangles, connecting their solutions to the problem posed (**MP.4**).

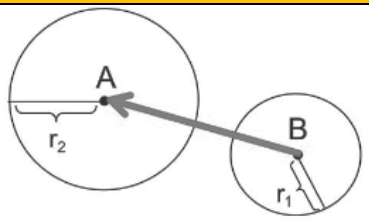
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply trigonometry to general triangles.	

Standards	Clarifications
KY.HS.G.13 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **MP.6**, **MP.7**	 Area of triangle = ½ ab sin(C)
KY.HS.G.14 (+) Understand and apply the Law of Sines and the Law of Cosines. a. Use the Law of Sines and Cosines to find unknown measurements in right and non-right triangles. b. Prove the Laws of Sines and Cosines and use them to solve problems. **MP.1**, **MP.3**	Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$ 

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Circles	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply theorems about circles.	

Standards	Clarifications
KY.HS.G.15 Verify using dilations that all circles are similar. **MP.5**, **MP.8**	
KY.HS.G.16 Identify and describe relationships among angles and segments within the context of circles involving: a. Recognize differences between and properties of inscribed, central and circumscribed angles. b. Understand relationships between inscribed angles and the diameter of a circle. c. Understand the relationship between the radius of a circle and the line drawn through the point of tangency on that radius. **MP.3, MP.5, MP.7**	Students recognize and apply relationships including the relationship between central, inscribed and circumscribed angles, inscribed angles on a diameter are right angles, the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
KY.HS.G.17 (+) Apply basic construction procedures within the context of a circle. a. Construct the inscribed and circumscribed circles of a triangle. b. Construct a tangent line from a point outside a given circle to the circle. **MP.5**, **MP.6**	Students build upon skills from other standards regarding construction procedures in the context of circles.

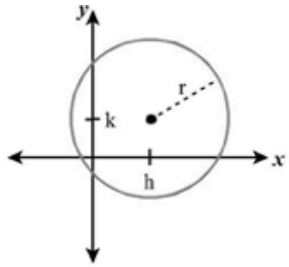
Attending to the Standards for Mathematical Practice
Students compare properties of a variety of circles to verify that all circles are similar (**MP.8**). Students use technology and drawings of circles to analyze properties of angles, radii and diameters that hold true across all circles (**MP.5**) and can explain these properties (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Circles	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Find arc lengths and areas of sectors of circles.	

Standards	Clarifications
KY.HS.G.18 (+) Understand the relationship between an intercepted arc length within a circle and the radius of the circle. a. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector. b. Define the radian measure of the angle as the measure of a central angle that intercepts an arc equal in length to the radius of the circle. **MP.2**, **MP.3**	$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{2\pi}$ $\frac{\text{Area of Sector}}{\pi r^2} = \frac{\text{Central Angle}}{2\pi}$ Area of Sector = $\frac{\text{Central Angle}}{2\pi} \bullet \pi r^2$ Area of Sector = $\frac{1}{2} \bullet$ Central Angle $\bullet r^2$

Geometry-Expressing Geometric Properties with Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Translate between the geometric description and the equation for a conic section.	

Standards	Clarifications
KY.HS.G.19 Understand the relationship between the algebraic form and the geometric representation of a circle. a. Write the equation of a circle of given center and radius using the Pythagorean Theorem. b. (+) Derive and write the equation of a circle of given center and radius using the Pythagorean Theorem. c. (+) Complete the square to find the center and radius of a circle given by an equation. **MP.6**, **MP.8**	 $(x - h)^2 + (y - k^2 = r^2$
KY.HS.G.20 (+) Derive the equations of conic sections. a. Derive the equation of a parabola given a focus and directrix. b. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **MP.2**, **MP.7**	Parabolas: $y - k = a(x - h)^2$ $x - h = a(y - k)^2$ Circles: $(x - h)^2 + (y - k)^2 = r^2$ Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Attending to the Standards for Mathematical Practice
Students explain the connection between the Pythagorean Theorem and the equation of a circle (**MP.8**) and use the center and radius accurately within the formula (**MP.6**).

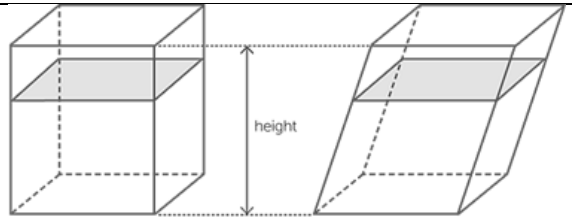
Geometry- Expressing Geometric Properties with Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use coordinates to prove simple geometric theorems algebraically.	

Standards	Clarifications
KY.HS.G.21 Use coordinates to justify and prove simple geometric theorems algebraically. **MP.2**, **MP.6**	Students understand how to prove or disprove a figure defined by four given points in the coordinate plane is a rectangle, as well as prove or disprove the given point lies on the circle centered at the origin and containing an additional given point.
KY.HS.G.22 Justify and apply the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. **MP.3**, **MP.7**	Students understand the relationship between slope and how it relates to both parallel and perpendicular lines. Within this standard, students also understand how to find the equation of a line parallel or perpendicular to a given line that passes through a given point.
KY.HS.G.23 Find measurements among points within the coordinate plane. a. Use points from the coordinate plane to find the coordinates of a midpoint of a line segment and the distance between the endpoints of a line segment. b. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **MP.2**, **MP.8**	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
KY.HS.G.24 Use coordinates within the coordinate plane to calculate measurements of two dimensional figures. a. Compute the perimeters of various polygons. b. Compute the areas of triangles, rectangles and other quadrilaterals.★ **MP.2**, **MP.4**	Students utilize the distance formula to find distances between points in order to find the area and/or perimeter of various geometric figures.

Attending to the Standards for Mathematical Practice
Students describe the connections between geometric theorems and their algebraic formulas (**MP.2**). They intentionally manipulate coordinates appropriately, fluently selecting criterion and formulas for a given context (**MP.7**). Students use coordinate geometry to model real-world situations, posing their own real-world problems when possible (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

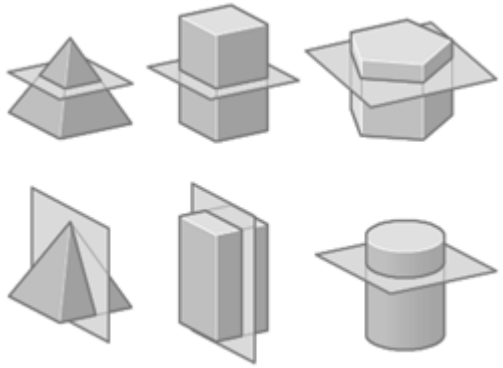
Geometry- Geometric Measurement and Dimensions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Explain volume formulas and use them to solve problems.	

Standards	Clarifications
KY.HS.G.25 Analyze and determine the validity of arguments for the formulas for the various figures and shapes. a. Finding the circumference and area of a circle. b. Finding the volume of a sphere, prism, cylinder, pyramid and cone. **MP.3**, **MP.7**	Students may use dissection arguments, Cavalieri's principle and informal limit arguments in order to find these values for these figures.
KY.HS.G.26 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **MP.2**, **MP.5**	
KY.HS.G.27 Use volume formulas to solve problems for cylinders, pyramids, cones, spheres, prisms ★ **MP.4**, **MP.6**	General Prism: $V = Bh$ Right Circular Cylinder: $V = \pi r^2 h$ Pyramid: $V = \frac{1}{3}Bh$ Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$

Attending to the Standards for Mathematical Practice
As students analyze volume formulas, they looking for relationships between the shapes and the related formulas (**MP.7**). Students critique different explanations or justifications for the formulas (**MP.3**). Students recognize various situations for which these formulas would apply and use them to solve real-world problems, posing their own real-world problems when possible (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Geometric Measurement and Dimensions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Visualize relationships between two-dimensional and three-dimensional objects.	

Standards	Clarifications
KY.HS.G.28 Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects. **MP.5**, **MP.7**	Students recognize visually the two dimensional shapes created via the cross sections of three dimensional solid figures. Examples include, but are not limited to 

Attending to the Standards for Mathematical Practice

Students use technology to identify the result of cutting a three- dimensional object and the result of rotating two-dimensional objects (**MP.5**). As students analyze two-dimensional and three-dimensional shapes, they gain insights into the structure of specific shapes (**MP.7**). For instance, students consider the two-dimensional figures that result from removing the top of a shoe box or from slicing an orange. Students compare and contrast the two-dimensional cross sections of an orange when sliced at different locations or angles verses slicing. For an extension, students can compare their conjectures from circles when slicing a cone at different locations or angles.

Geometry-Modeling with Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply geometric concepts in modeling situations.	

Standards	Clarification/Illustration
KY.HS.G.29 Use geometric shapes, their measures and their properties to describe objects in real world settings. **MP.1**, **MP.4**	Students use geometric shapes to model objects, for example, modeling a tree trunk or a human torso as a cylinder).★
KY.HS.G.30 Apply concepts of density based on area and volume in modeling situations, using appropriate units of measurement. **MP.4**, **MP.6**	Students explore scenarios where they find the area of regions and the volume of solid figures. In the process, they appropriately use units of measurement, for example, persons per square mile, BTUs per cubic foot
KY.HS.G.31 Apply geometric methods to solve design problems. ★ **MP.1**, **MP.4**	Students practice modeling techniques in this standard using a variety of strategies and practices, for example, designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios

Attending to the Standards for Mathematical Practice

Students recognize various situations for which geometric knowledge would apply and do so to solve real-world problems (**MP.4**). As students use geometric methods to solve design problems, they continually reflect on whether their method and process makes sense for the problem and revise, as needed, until a viable solution has been found (**MP.1**)**.** Students also select appropriate theorems and formulas and report units with appropriate accuracy (**MP.6**).

Kentucky Academic Standards for Mathematics: Conceptual Category Statistics and Probability★

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data	Making Inferences and Justifying Conclusions	Conditional Probability and the Rules of Probability	Using Probability to Make Decisions
• Summarize, represent and interpret data on a single count or measurement variable. • Summarize, represent and interpret data on two categorical and quantitative variables. • Interpret linear models.	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments and observational studies.	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model.	• Calculate expected values and use them to solve problems. • Use probability to evaluate outcomes of decisions.

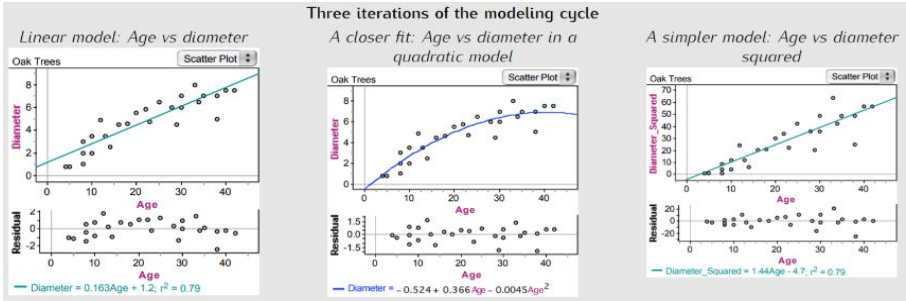
Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Statistics and Probability-Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Summarize, represent and interpret data on a single count or measurement variable.	
Standards	**Clarifications**
KY.HS.SP.1 Represent the distribution of data with plots on the real number line (stem plots, dot plots, histograms and box plots). **MP.4**, **MP.5**	Students create appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.2 Use statistics appropriate to the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. **MP.2**, **MP.6**	Students use raw data and data from appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.3 Interpret differences in shape, center and spread in the context of the distributions of the numerical data, accounting for the presence and possible effects of extreme data points (outliers). **MP.1**, **MP.7**	Students analyze contextual situations as they interpret differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.4 (+) When appropriate, fit a normal distribution to a numerical data set for given mean and standard deviation and then estimate population percentages using the Empirical Rule and recognize that there are data sets for which such a procedure is not appropriate. **MP.1**, **MP.3**	Students use the empirical rule (68%-95%-99.7% rule), calculators and/or tables to estimate areas under the normal curve, recognizing when data sets are skewed this can be problematic.
Attending to the Standards for Mathematical Practice	
Students use technology to visualize data using stem plots, dot plots, histograms and box plots (**MP.5**). After the data have been collected, students are precise about choosing the appropriate analyses and representations to reveal the variability in the data (**MP.6**). Students analyze quantitative data and classify any observation(s) that deviate(s) considerably from the majority of data within a distribution as potential outliers (**MP. 7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability- Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Summarize**, **represent and interpret data on two categorical and quantitative variables.**	

Standards	Clarifications
KY.HS.SP.5 Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal and conditional relative frequencies (probabilities) in the context of the data, recognizing possible associations and trends in the data. **MP.2**, **MP.7**	Students use frequency tables to both calculate probabilities, as well as determine relationships between the variables represented in those tables.
KY.HS.SP.6 Represent data on two quantitative variables on a scatter plot and describe how the explanatory and response variables are related. a. Calculate an appropriate mathematical model, or use a given mathematical model, for data to solve problems in context. b. Informally assess the fit of a model (through calculating correlation for linear data, plotting, calculating and/or analyzing residuals). **MP.3**, **MP.4**, **MP.5**	Emphasize linear, quadratic and exponential models as illustrated below. 

Attending to the Standards for Mathematical Practice
Students discover structures or patterns in data to answer statistical questions using tables or appropriate representations (**MP.7**). Students informally determine whether a selected model is appropriate for a set of data and use technology when appropriate to do so (**MP 5**). Students draw and discuss conclusions about a statistical question (**MP.3**) using appropriate mathematical models.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability-Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret linear models.	

Standards	Clarifications
KY.HS.SP.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **MP.1**, **MP.2**	Students demonstrate interpreting slope in the context of a given situation when examining two variable statistics as being "for each additional known unit increase in an explanatory variable, we expect or predict a known unit increase (or decrease) in the response variable." Students demonstrate interpreting intercept in the context of a given situation when examining two variable statistics as being "the predicted known unit of a response variable when the explanatory variable is zero known units."
KY.HS.SP.8 Understand the role and purpose of correlation in linear regression. a. Use technology to compute correlation coefficient of a linear fit. b. Interpret the meaning of the correlation within the context of the data. c. Describe the limitations of correlation when establishing causation. **MP.5**, **MP.6**	a. Students use technology to perform the calculation of: $$r = \frac{\Sigma(x - \overline{x})}{\sqrt{\Sigma(x - \overline{x})^2}\sqrt{\Sigma(y - \overline{y})^2}}$$ b. Students understand correlation measures linear associations between two quantitative variables addressing the direction (positive or negative) and the relative strength of the given association. c. Students understand one of the most common misinterpretations of correlation is to think of it as a synonym for causation. A high correlation between two variables (suggesting a statistical association between the two) does not imply one causes the other.

Attending to the Standards for Mathematical Practice

Students interpret the results to a statistical question and relate the results to the context of the data (**MP.1**, **MP.2**). Students use technology to compute correlation coefficients (**MP.5**). Students recognize that correlation is an indication of a linear relationship between two quantitative variables and not simply another word for association (**MP.6**).

Statistics and Probability-Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and evaluate random processes underlying statistical experiments.	

Standards	Clarifications
KY.HS.SP.9 Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. **MP.1**, **MP.3**	Students use sample statistics (mean and standard deviation) obtained from random samples to help estimate population parameters which are unknown values.
KY.HS.SP.10 Decide if a specified model is consistent with the results from a simulation. **MP.3**, **MP.6**	If a model shows a spinning coin falls heads-up with probability of 0.5, would a result of 5 tails in a row cause you to question the model?

Attending to the Standards for Mathematical Practice
Students follow the progression of the statistical problem-solving process to investigate answers to a statistical question (**MP.3**)**.** Students justify their conclusions, communicate them to others (orally and in writing) and critique the conclusions of others (**MP.3**)**.** Students are precise about choosing the appropriate analyses and representations that account for the variability in the data (**MP.6**)**.**

Statistics and Probability- Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Make inferences and justify conclusions from sample surveys, experiments and observational studies.	

Standards	Clarifications
KY.HS.SP.11 Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **MP.3**, **MP.8**	Students understand a random selection of 100 students from your school will allow you to draw some conclusions about all the students in the school, whereas taking your class as a sample will not allow that generalization. Students recognize experiments involve imposing treatments on units/subjects, whereas observational studies do not.
KY.HS.SP.12 Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. **MP.4**, **MP.7**	KY.HS.SP.12 differs from KY.HS.SP.9 in that results from non-random samples (Voluntary Response and Convenience) generate biased results when compared with more appropriate, random samples of the same population.
KY.HS.SP.13 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. **MP.3**, **MP.8**	Hypotheses can be tested to determine if significant differences between two treatments exist using statistical data. If significance exists, claims and conclusions can be made about the treatment.

Attending to the Standards for Mathematical Practice
Students compare and contrast the different roles randomization plays in data collection (**MP.8**)**.** Students look for patterns in the variability around the structure (**MP.7**)**.**

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand independence and conditional probability and use them to interpret data.	

Standards	Clarifications
KY.HS.SP.14 Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as, • as unions, "A or B," that are mutually exclusive events and • as unions, "A or B," that are non-mutually exclusive events and • as intersections, "A and B," and • as complements of other events, "not A." to calculate basic probabilities. **MP.1**, **MP.2**	A union of two events, "A or B," includes *all* elements in both events notated by: $A \cup B$. Addition Rule for mutually exclusive events: If A and B are mutually exclusive, P(A or B) = P(A) + P(B). Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. An intersection, "A and B," of two events includes all overlapping elements notated by: $A \cap B$. A complement for any event A, P(not A) = 1 – P(A).
KY.HS.SP.15 Understand the concept of independence. a. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their individual probabilities, P(A) x P(B) b. (+) Determine whether two events are independent and provide a justification to support the decision. c. Recognize and explain the concept of independence in everyday language and everyday situations. **MP.1**, **MP.6**	a. Events *A* and *B* are independent if and only if P(*A* and *B*) = *P*(*A*)*P*(*B*).
KY.HS.SP.16 Understand the concept of conditional probability. a. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*). b. (+) Interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A* and the conditional probability of *B* given *A* is the same as the probability of *B*.	a. For any two events *A* and *B,* P(*A* given *B*) = $\frac{P(A \text{ and } B)}{P(A)}$.

c. Recognize and explain the concept of conditional probability in everyday language and everyday situations. d. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A and* interpret the answer in terms of the model. **MP.1**, **MP.3**	
KY.HS.SP.17 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. **MP.2**, **MP.4**	Students collect their own data or use data obtained from a random sample of their choosing and construct two-way frequency tables from their sample in order to determine independence and calculate probabilities.

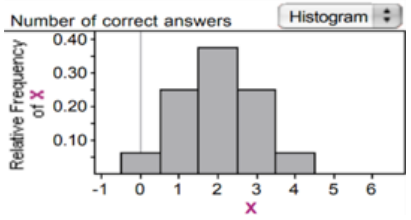
Attending to the Standards for Mathematical Practice
Students encounter chance events in real contexts, including situations involving both dependent and independent events, are able to determine the difference between the contexts and fluently select and use appropriate formulas (**MP.1**). Students consider whether the occurrence of one event affects the probability of the other event (**MP.2**) in order to determine if two events are independent. Students analyze and discuss a variety of sources such as contingency tables to provide a context for conditional probability (**MP.3**). Students consider how conditions or assumptions affect the computation of a probability (**MP.6**).

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use the rules of probability to compute probabilities of compound events	

Standards	Clarifications
KY.HS.SP.18 (+) Apply the General Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B), in a uniform probability model and interpret the answer in terms of the model. **MP.1**, **MP.2**	Consider an experiment where two cards are drawn without replacement. Define events A and B: A = 1st card drawn is a king B = 2nd card drawn is a king P(B|A) is the probability that the second card is a king given the first card drawn was a king. In that case, there will be 3 kings left out of 51 cards, so P(B|A) = 3/51 P(A and B) = P(1st is a king and 2nd is a king) P(A and B) = P(1st king) • P(2nd is a king, given 1st is a king) P(A and B) = (4/52) (3/51) P(A and B) = P(A) • P(B|A)
KY.HS.SP.19 Use permutations and combinations to compute probabilities. a. Distinguish between situations that can be modeled using counting techniques, including Fundamental Counting Principle, permutations and combinations. b. Perform calculations using the appropriate counting technique, including simple probabilities. c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **MP.1**, **MP.8**	Permutations are calculated when order matters. Combinations are calculated when order does not matter. Number of permutations of *n* items taken *r* at a time: ${}_nP_r = \frac{n!}{(n-r)!}$ Number of combinations of *n* items taken *r* at a time: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Attending to the Standards for Mathematical Practice
Students recognize and solve real-world problems using the Fundamental Counting Principle, Permutations and Combinations (**MP.1**)**.** Students identify patterns to generalize a formula for calculating permutations and combinations (**MP.8**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability-Using Probability to Make Decisions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Calculate expected values and use them to solve problems.	
Standards	**Clarifications**
KY.HS.SP.20 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays as for data distributions. **MP.3**, **MP.6**	Students realize random variables are different from the variables used in other high school domains and random variables are functions of the outcomes of a random process and thus have probabilities attached to their possible values. A possible example of a probability distribution: **Number Correct (x) / Probability** 0: $\frac{1}{16}$ 1: $\frac{4}{16}$ 2: $\frac{6}{16}$ 3: $\frac{4}{16}$ 4: $\frac{1}{16}$ 
KY.HS.SP.21 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions. **MP.1**, **MP.8**	The expected value/mean of a discrete random variable is $\mu = E(x) = \Sigma xp(x)$.
KY.HS.SP.22 (+) Develop a probability distribution for a random variable. a. Find an expected value based on a sample space in which theoretical probabilities can be calculated. b. Find an expected value based on a sample space in which empirical probabilities can be calculated. **MP.2**, **MP.8**	a. (+) Theoretical probability is given by the number of ways a particular event can occur divided by the total number of possible outcomes. b. (+) The empirical probability of an event is given by number of times an event occurs divided by the total number of incidents observed.

Statistics and Probability-Using Probability to Make Decisions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use probability to evaluate outcomes of decisions.	
Standards	**Clarifications**
KY.HS.SP.23 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values. c. Use calculated expected values to make fair decisions and formulate strategies. **MP.6**, **MP.8**	Students use expected values to play a role in decision making in a variety of contexts.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Calculus (+)

Calculus instructional time should focus on 3 critical areas:

1. Conceptual understanding and procedural fluency of limits, derivatives and integration.
2. Applications of derivatives and integrals.
3. Working with functions in a variety of ways: graphical, numerical, analytical and verbal.

Calculus Overview

Limits	Function Behavior	Continuity	Understanding the Derivative	Applications of the Derivative	Understanding Integration	Applications of Integration
• Understanding the concept of the limit of a function.	• Describe the asymptotic and unbounded behavior of functions.	• Develop an understanding of continuity as a property of functions.	• Demonstrate an understanding of the derivative.	• Apply differentiation techniques.	• Understand and apply the Fundamental Theorem of Calculus.	• Apply techniques of integration. • Use integration to solve problems.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

(+): Calculus standards are not required standards for all Kentucky students; therefore, all Calculus standards would be considered (+) standards.

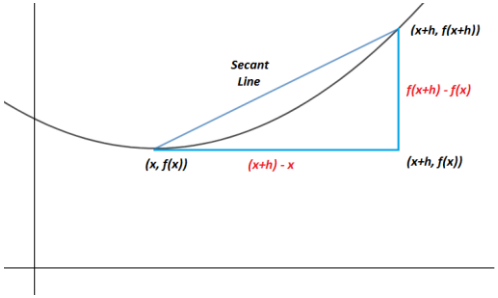
TWO plus signs (++) indicate a standard that is optional even for calculus.

Calculus-Limits	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the concept of the limit of a function.	
Standards	**Clarifications**
KY.HS.C.1 (+) Understand limits. a. Apply limits to a variety of functions, including piecewise functions. b. (++) Prove that the limit of a function exists, based upon the definition of a limit. **MP.2, MP.3**	
KY.HS.C.2 (+) Demonstrate an understanding of limits by estimating and finding the limit of a function at a point graphically, numerically and algebraically. **MP.5, MP.8**	Include analysis of limits in piecewise functions. Algebraic techniques include but are not limited to factoring, multiplying by the conjugate and finding the lowest common denominator.
KY.HS.C.3 (+) Apply properties and theorems of limits, including limits of indeterminate forms. **MP.2, MP.3**	Include sums, differences, products, quotients, composition of functions, special limits, Squeeze Theorem and L'Hospital's Rule.
KY.HS.C.4 (+) Communicate understanding of limits using precise mathematical symbols and language. **MP.3, MP.6**	Use of limits to predict the function value for an undefined value (hole in the graph). Apply the definition of a limit to margin of error. For example, if the weight of a golf ball needs to be within a certain range (ϵ), then the radius of the ball must be to a certain level of accuracy (δ).
Attending to the Standards for Mathematical Practice	
Students can use technology to examine the graph of a function and determine whether or not the limit of the function exists at a point (**MP.5**). Students can use a table to find the value of a function for points that approach a given point, leading to conjectures about the limit of the function (**MP.8**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Function Behavior	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Describe the asymptotic and unbounded behavior of functions.	
Standards	**Clarifications**
KY.HS.C.5 (+) Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **MP.2, MP.5**	
KY.HS.C.6 (+) Discuss the end behavior of functions; identify representative functions for each type of end behavior using precise mathematical symbols and language. **MP.2, MP.6**	$\lim_{x\to\infty} f(x) = 4$ implies a horizontal asymptote of *y* = 4 $\lim_{x\to\infty} f(x) = \infty$ implies right hand end behavior is positive infinity NOTE: odd functions result in end behavior similar to lines (opposite directions); even functions result in end behavior similar to parabolas (same direction)
Attending to the Standards for Mathematical Practice	
Students use analytic methods to identify vertical and horizontal asymptotes (**MP.2**). Students use technology to examine the graph of a function, to determine the values for which it is defined and to convergence for increasingly large values in the domain (**MP.5**).	

Calculus-Continuity	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Develop an understanding of continuity as a property of functions.	
Standards	**Clarifications**
KY.HS.C.7 (+) Understand and use the limit definition of continuity to determine whether a given function is continuous at a specific point. **MP.2, MP.3**	If a function is continuous at $x = c$, then $\lim_{x \to c} f(x) = f(c)$.
KY.HS.C.8 (+) Define and identify different types of discontinuity – removable (hole) or non-removable (jump, asymptote) – in terms of limits. **MP.3, MP.6**	Non-removable discontinuity is identified by vertical asymptotes (infinite discontinuity) and jumps (non-agreement of left- and right-hand limits). Removable discontinuity is represented by a hole in the graph (agreement of left- and right-hand limits). Include analysis of special limits, such as $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
KY.HS.C.9 (+) Understand and apply continuous function theorems. a. Apply the Intermediate Value Theorem to continuous functions. b. Apply the Extreme Value Theorem to continuous functions. **MP.2, MP.3**	a. Intermediate Value Theorem illustration: Sarah's mom measures her height every year on her birthday. On her 10th birthday, Sarah was 48 inches tall and on her 11th birthday she measured 52 inches. Her cousin said, "You were the same height as me sometime during this year." How tall is Sarah's cousin? Justify your answer. b. The Extreme Value Theorem is contingent on the concept of continuity, but will not be addressed in sequence until the concept of derivatives and critical numbers is established.
KY.HS.C.10 (+) Communicate an understanding of continuity using precise mathematical symbols and language. **MP.2, MP.6**	Continuity on a closed interval $[a, b]$ requires continuity on the open interval (a, b), $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$
Attending to the Standards for Mathematical Practice	
Students explain why a function is continuous or continuous at a point or over an interval (**MP.3**). Students use technology to examine the graph of a function and determine whether it is continuous in a given interval (**MP.5**).	

Calculus-Understanding the Derivative	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Demonstrate an understanding of the derivative.	
Standards	**Clarifications**
KY.HS.C.11 (+) Define derivatives. a. Define the derivative of a function as the limit of the difference quotient. b. Understand this limit of the difference quotient can be interpreted as an instantaneous rate of change or the slope of a tangent line. **MP.5, MP.8**	The difference quotient $\frac{f(x+h)-f(x)}{h}$ represents the slope of the secant line between $(x, f(x))$ and $(x+h, f(x+h))$ as shown below. The secant line approaches the tangent line as *h* approaches 0. 
KY.HS.C.12 (+) Use average rate of change to estimate the derivative from a table of values or a graph. **MP.2, MP.8**	
KY.HS.C.13 (+) Understand the derivative as a function. **MP.2, MP.5**	Analysis of the derivative as a function implies characteristics of the original function. This analysis can be done analytically or graphically. Students move fluently between graphs of functions and derivatives using each to predict what the other would look like. (For example, positive/negative values of derivative imply increasing/decreasing of original function.) Differentiability of original function implies the derivative is a continuous function.
KY.HS.C.14 (+) Apply the definition of derivative to find derivative values and derivative functions.	Include the formal definition of a derivative: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

MP.2, MP.3	The alternate form of this formal definition is to calculate the derivative at one particular value.
KY.HS.C.15 (+) Explain why differentiability implies continuity yet continuity does not imply differentiability. **MP.3, MP.6**	
KY.HS.C.16 (+) Understand and apply the Mean Value Theorem, including numerical, graphical and algebraic representations. **MP.2, MP.5**	
KY.HS.C.17 (+) Understand the relationship between the concavity of a function and the sign of the second derivative. **MP.2, MP.3**	
KY.HS.C.18 (++) Understand Rolle's Theorem as a special case of the Mean Value Theorem. **MP.2, MP.3**	

Attending to the Standards for Mathematical Practice
Students use analytic strategies to determine characteristics of functions as they relate to derivatives (**MP.2**) and technology to confirm the analytic results (**MP.5**). Students use tables of values to examine the average rate of change of a function over smaller and smaller intervals, leading to the derivative as the instantaneous rate of change (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Derivatives	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply differentiation techniques.	

Standards	Clarifications
KY.HS.C.19 (+) Efficiently find derivatives of functions with and without technology. **MP.2, MP.5**	Functions include linear, quadratic, polynomial, exponential, logarithmic (including bases other than e), trigonometric (including inverses), square root and other root functions. Efficiently finding a derivative involves selecting the most appropriate formula. For example, the derivative of $f(x) = x/4$ can be found using the quotient rule, but it is more efficient to use the power rule to find the derivative of $f(x) = \frac{1}{4}x$.
KY.HS.C.20 (+) Understand and use derivative rules for sums, differences, products and quotients of two functions and calculate the derivative of a composite function using the chain rule. **MP2, MP.3**	
KY.HS.C.21 (+) Use implicit differentiation to find a derivative in an equation of two variables. **MP.1, MP.2**	Include a variety of functions (such as polynomial, root, logarithmic, exponential and trigonometric. Implicit differentiation can be used to explore rules such as exponential and logarithmic for bases other than e.
KY.HS.C.22 (+) Use implicit differentiation to find the derivative of the inverse of a function. **MP.2, MP.3**	
KY.HS.C.23 (+) Understand the relationship between the increasing and decreasing behavior of a function and the sign of the first derivative of the function. **MP.1, MP.2**	

KY.HS.C.24 (+) Use the first derivative to analyze curves and identify relative extrema. **MP.2, MP.3**	The Extreme Value Theorem is useful in optimization problems involving closed intervals since absolute extrema may occur at endpoints. (For example, consider using wire to create a circle and square of maximum value. The maximum area is obtained by using all the wire on the circle).
KY.HS.C.25 (+) Understand the relationship of concavity to the second derivative. **MP.2, MP.5**	
KY.HS.C.26 (+) Use the second derivative to find points of inflection. **MP.2, MP.3**	Points of inflection must be defined values for the function.
KY.HS.C.27 (+) Use the second derivative to analytically locate intervals on which a function is concave up, concave down or neither. **MP.2, MP.3**	
KY.HS.C.28 (+) Describe how graphical characteristics of a given function, the first derivative of that function and the second derivative of that function interrelate. **MP.2, MP.5**	
KY.HS.C.29 (+) Use derivatives to express rate of change in a variety of contexts. **MP.2, MP.4**	Examples include but are not limited to exponential growth (population) and decay (half-life), logistic growth, continuous interest and Newton's Law of Cooling.
KY.HS.C.30 (+) Use derivatives to solve a variety of problems including related rates, optimization, tangent line approximations and growth and decay models. **MP.1, MP.4**	Related rate examples include but are not limited to relating variables using the Pythagorean Theorem, relating variables using trigonometric relationships and relating variables using geometric formulas. Tangent line approximations (linearization): ● Tangent lines make good approximations of function values close to the point of tangency. ● Tangent line approximations will be an overestimate if the function is concave down. ● Tangent line approximations will be an underestimate if the function is concave up. Growth and Decay: ● Use the derivative to calculate the rate of change of growth or decay at a specific time.

KY.HS.C.31 (+) Use differentiation to solve problems involving velocity, speed and acceleration. **MP.1, MP.2**	
KY.HS.C.32 (+) Understand and apply differential equations. a. Verify solutions to differential equations and use them to model real-world problems with and without technology. b. Solve separable differential equations and use them in modeling real-world problems with and without technology. **MP.1, MP.4**	Solving separable equations requires integration, however, students establish patterns for recognizing what makes a solution work. ● Students create differential equations by starting with the answers ● Students understand what makes the differential equation separable.

Attending to the Standards for Mathematical Practice
Students use derivatives to identify and describe the characteristics of a function (**MP.2, MP.6**)**.** Contextual questions about optimal or extreme values can be identified by representing problem situations in a variety of ways (**MP.4**) and applying appropriate tools and techniques to solve the questions that are posed (**MP.1**).

Calculus-Understanding Integration	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Demonstrate understanding of a definite integral.	

Standards	Clarifications
KY.HS.C.33 (+) Understand the definite integral of a function over an interval. Interpret a definite integral as a limit of Riemann Sums and as net accumulation of change. **MP.2, MP.5**	
KY.HS.C.34 (+) Write a Riemann sum that represents the definition of a definite integral. **MP.2, MP.3**	
KY.HS.C.35 (+) Calculate the values of Riemann Sums over equal subdivisions to approximate definite integrals of functions represented graphically and numerically (using tables). Use left-hand sums, right-hand sums, midpoint sums and trapezoidal sums. **MP.2, MP.3**	For strictly increasing functions, a right-hand sum overestimates and left-hand sum underestimates. For strictly decreasing, the opposite is true.
KY.HS.C.36 (+) Recognize differentiation and integration as inverse operations. **MP.2, MP.8**	Integration rules can be established by reversing derivative rules. Many integration rules can be developed using implicit derivatives and/or substitution.
KY.HS.C.37 (+) Understand how the Fundamental Theorem of Calculus connects differentiation and integration and use it to evaluate definite and indefinite integrals and to represent particular antiderivatives. **MP.2, MP.3**	Include understanding and applying the Second Fundamental Theorem of Calculus.
KY.HS.C.38 (+) Perform analytical and graphical analysis of functions using the Fundamental Theorem of Calculus. **MP.2, MP.5**	Use integration capabilities of graphing utilities to verify solutions obtained by applying the Fundamental Theorem of Calculus.

KY.HS.C.39 (+) Understand and use the definite integral of a function over an interval and understand its use as a mathematical tool. **MP.1, MP.2**	

Attending to the Standards for Mathematical Practice
Students understand how graphical displays of functions (**MP.5**) and the application of limits (**MP.2**) lead to the concept of integration.

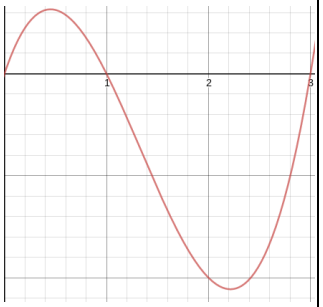
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Integration	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply techniques of integration.	

Standards	Clarifications
KY.HS.C.40 (+) Find antiderivatives of a variety of basic functions including power, exponential, logarithmic and trigonometric and apply basic properties of definite integrals. **MP.2, MP.7**	
KY.HS.C.41 (+) Use substitution techniques and change of limits of integration to find antiderivatives. **MP.2, MP.3**	Combining substitution techniques with basic rules allows for a broad spectrum of additional functions to be integrated. Substitution is the derivative equivalent of the chain rule and may be used to develop basic integration rules.
KY.HS.C.42 (+) Find particular antiderivatives given initial conditions. **MP.1, MP.2**	

Attending to the Standards for Mathematical Practice
When applying techniques of integration represent problem situations (**MP.1**)**,** students identify whether an available techniques (**MP.7**) is applicable to a given integral expression (**MP.2**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Integration	
Standards for Mathematical Practice	
MP.1. Make Sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Define trigonometric ratios and solve problems involving right triangles.	
Standards	**Clarifications/Illustrations**
KY.HS.C.43 (+) Model, solve and interpret applications of antiderivatives including finding area, velocity, acceleration and volume of a solid. **MP.1, MP.4**	Include area under a curve and area between two curves. Students calculate intersection points and note when functions "switch" requiring two integrals. Students calculate horizontal area. Students calculate volume using the dish, washer and shell methods.
KY.HS.C.44 (+) Apply integration to solve problems including particle motion and exponential growth and decay. **MP.1, MP.4**	Include particle motion problems, such as the velocity function below. • Where does particle change direction? • When is it moving to the left? • When is it moving to the right? • How far does it move to the left? • How far does it move to the right? • What is the displacement of the particle? • What is the total distance traveled? • If the particle started at x = 5, where is it at the end of the first 3 seconds? 
KY.HS.C.45 (+) Describe the application of integration to a variety of problems using precise mathematical language and notation. **MP.4, MP.6**	Use definite integrals to represent displacement, total distance traveled and average value of a function. Integrals are solutions to differential equations, such as $\frac{dy}{dx} = ky$ is the solution to $y = Ce^{kt}$.
Attending to the Standards for Mathematical Practice	
Students recognize that a variety of applied problems can be represented using integral expressions (**MP.4**) and identify appropriate integration strategies (**MP.2**) to solve these problems (**MP.1**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1

Common Addition and Subtraction Situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	Group Size Unknown	Number of Groups Unknown
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3
Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\,x\,(b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$
If $a > b$ and $b > c$ then $a > c$
If $a > b$, then $b < a$
If $a > b$, then $-a < -b$
If $a > b$, then $a \pm c > b \pm c$
If $a > b$ and $c > 0$, then $a \times c > b \times c$
If $a > b$ and $c < 0$, then $a \times c < b \times c$
If $a > b$ and $c > 0$, then $a \div c > b \div c$
If $a > b$ and $c < 0$, then $a \div c < b \div c$

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★