FCPS Pre-Calculus Standards (The "PLUS" Standards)

The following **K**entucky **A**cademic **S**tandards are to be taught in Pre-Calculus courses.

Number and Quantity

The Complex Number System

- Perform arithmetic operations with complex numbers

KY.HS.N.7 Understanding properties of complex numbers. a. Know there is a complex number i such that $i^2 = -1$ and every complex number has the form $a + bi$ with a and b real. b. Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. **MP.7, MP.8**	An important difference between rational and irrational numbers is that rational numbers form a number system. Students understand that if you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Students also understand that multiplying the irrational number 2 by itself, yields a rational number, 2. Irrational numbers are defined by not being rational and this definition can be exploited to generate many examples of irrational numbers from just a few.

- Represent complex numbers and their operations on the complex plane

KY.HS.N.8 (+) Understanding representations of complex numbers using the complex plane. a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. b. Represent addition, subtraction, multiplication, modulus and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. **MP.2, MP.5**	a. Students graph in both rectangular and polar form and convert rectangular coordinates to polar coordinates and vice versa. Students understand this conversion preserves the equality of the two forms. c. Students understand that calculating the distance between numbers in the complex plane is fundamentally the same as calculating distances in the standard coordinate plane using the distance formula from grade 8. Students understand calculating the midpoint of a segment in the complex plane as the average of the a values and average of the b values in any two endpoints expressed as a +bi.

- Use complex numbers in polynomial identities and equations

KY.HS.N.10 (+) Extend polynomial identities to the complex numbers. **MP.7, MP.8**	When multiplying complex binomials, students recognize and understand the value of i^2 as -1 and fluently simplify each polynomial appropriately navigating between the real number system and complex numbers. One example of this might be that students should understand that it would be appropriate to rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
KY.HS.N.11 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **MP.1, MP.3**	

Vector and Matrix Quantities

- Represent a model with vector quantities

KY.HS.N.12 (+) Understand and apply properties of vectors. a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. c. Solve problems involving velocity and other quantities that can be represented by vectors. **MP.1. MP.6**	a. Vectors are directed by an angle and continue in that direction for a set length. b. Students connect 1) finding vertical and horizontal components and the magnitude of a vector with 2) using the Pythagorean Theorem in the coordinate plane. Limit to two-dimensional vectors.

- Perform operations on vectors

KY.HS.N.13 (+) Perform operations with vectors (addition, subtraction and multiplication by a scalar). a. Add vectors end-to-end, component-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise. d. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. e. b. Compute the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). **MP.3, MP.7**	

Algebra

Arithmetic with Polynomials and Rational Expressions

- Use polynomial identities to solve problems

KY.HS.A.6 (+) Know and apply the Remainder Theorem. **MP.1, MP.8**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
KY.HS.A.8 (+) Prove polynomial identities and use them to describe numerical relationships. **MP.2, MP.3, MP.6**	Students observe the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

- Rewrite rational expressions

KY.HS.A.10 (+) Rewrite simple rational expressions in different forms. **MP.7, MP.8** KY.HS.A.11 (+) Add, subtract, multiply and divide rational algebraic expressions. **MP.2, MP.3**	Students observe how to write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x) and r(x) are polynomials with the degree of r(x) less than the degree of b(x). Methods of rewriting rational expressions could include, but are not limited to: • Inspection • Synthetic division • Long division • Use of technology Students go beyond demonstrating procedural fluency and apply this standard in a variety of contextual situations.

Functions

Interpreting Functions

- Analyze functions using different representations

KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★ a. Graph linear and quadratic functions and show intercepts, maxima and minima. b. Graph square root, cube root and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior. e. (+) Graph trigonometric functions, showing period, midline and amplitude. f. (+) Graph piecewise functions, including step functions. g. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. **MP.4, MP.5**	Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features important for identifying and describing functions. This standard indicates the function families in students' repertoires, detailing which features are required for several key families. Students demonstrate fluency with linear, quadratic and exponential functions, including the ability to graph without using technology. In other function families, students graph simple cases without technology and more complex ones with technology.

Building Functions

- Build a function that models a relationship between two quantities

KY.HS.F.6 Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions. **MP.4, MP.7**	c. Consider contextual examples for composition functions, such as, if T(y) is the temperature in the atmosphere as a function of height and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

- Build new functions from existing functions

KY.HS.F.9 Find inverse functions. a. Given the equation of an invertible function, find the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2, MP.6**	b-d. Students need a formal sense of inverse functions. Students understand a function and its inverse describe the exact same relationship but in different ways.
KY.HS.F.10 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents with the use of technology. **MP.1, MP.7**	Students can use inverses of simple logarithmic and exponential equations in order to solve those equations. The inverse relationship between logarithmic and exponential functions is special in that each function's inverse is also a function
Trigonometric Functions	
● Extend the domain of trigonometric functions using the unit circle	
KY.HS.F.15 (+) Understand the relationship of radian measure of an angle to its arc length. **MP.1, MP.6**	Understanding **radian measure of an angle as arc length on the unit circle** enables students to build on their understanding of trigonometric ratios associated with acute angles and to explain how these ratios extend to trigonometric functions whose domains are included in the real numbers.
KY.HS.F.16 (+) Understand and use the unit circle. a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. b. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$ and use the unit circle to express the values of sine, cosine and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x, where x is any real number. c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MP.7, MP.8**	This standard is sometimes called "unwrapping the unit circle." For each function, the angle θ is represented by values on the horizontal axis and the resulting outputs are graphed on the vertical axis. c. Students understand symmetry exists within the unit circle for paired reference angles: $\sin(-\theta) = -\sin(\theta)$, so sine is an odd function; and $\cos(-\theta) = \cos(\theta)$, so cosine is an even function.
● Model periodic phenomena with trigonometric functions	
KY.HS.F.17 (+) Choose trigonometric functions to model periodic phenomena with specified period, midline and amplitude. ★ **MP.4, MP.5, MP.6**	A function is described as sinusoidal or is called a sinusoid if it has the same shape as the sine graph, for example, has the form □(□) = □ + □(sin□ +□). Many real-world phenomena can be approximated by sinusoids, including sound waves, oscillation on a spring, the motion of a pendulum, tides and phases of the moon. Because sin(□)oscillates between 1 and 1, □ + □(sin□ +□)will oscillate between □ − □ and □ + □. Thus, □= □ is the midline and □ is the amplitude of the sinusoid. Students can obtain the frequency of □: the period

	of sin(□) is 2□, so (knowing the effect of multiplying □ by □) the period of sin(□) is 2□/□ and the frequency is its reciprocal. When modeling, students have the sense that □ affects the frequency and that □ and □ together produce a phase shift, but finding a correct solution might involve technological support, except in simple cases.
KY.HS.F.18 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MP.2, MP.3**	Students experience restricting the domain of a function so it has an inverse. For trigonometric functions, a common approach to restricting the domain is to choose an interval on which the function is always increasing or always decreasing.
KY.HS.F.19 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. ★ **MP.4, MP.5**	Include sin⁻¹□,cos⁻¹□, and tan⁻¹□.

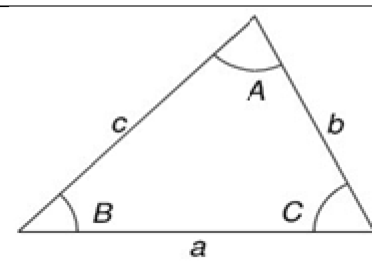
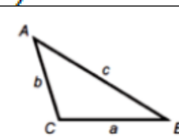
- Prove and apply trigonometric identities

KY.HS.F.20 (+) Proving identities and formulas within the context of trigonometry. a. Prove the Pythagorean identity and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. b. Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems. **MP.3, MP.7**	In the unit circle, the x-value is the cosine and the y-value represents the sine. Since the hypotenuse of any right triangle on the unit circle is 1, the Pythagorean relationship of □²+□²=1 holds. Students connect the Pythagorean Theorem in geometry and the study of trigonometry to understand this relationship.

Geometry

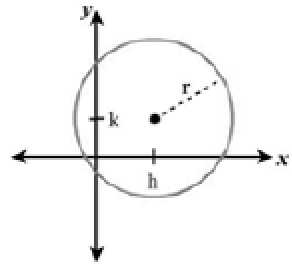
Similarity, Right Triangles and Trigonometry

- Apply trigonometry to general triangles

KY.HS.G.13 (+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **MP.6, MP.7**	 **Area of triangle = ½ ab sin(C)**
KY.HS.G.14 (+) Understand and apply the Law of Sines and the Law of Cosines. a. Use the Law of Sines and Cosines to find unknown measurements in right and non-right triangles. b. Prove the Laws of Sines and Cosines and use them to solve problems. **MP.1, MP.3**	Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$ 

Expressing Geometric Properties and Equations

- Translate between the geometric description and the equation of a conic section

KY.HS.G.19 Understand the relationship between the algebraic form and the geometric representation of a circle. a. Write the equation of a circle of given center and radius using the Pythagorean Theorem. b. (+) Derive and write the equation of a circle of given center and radius using the Pythagorean Theorem. c. (+) Complete the square to find the center and radius of a circle given by an equation. **MP.6, MP.8**	 $(x-h)^2 + (y-k^{2} = r^2$
KY.HS.G.20 (+) Derive the equations of conic sections. a. Derive the equation of a parabola given a focus and directrix. b. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **MP.2, MP.7**	Parabolas: $y-k = a(x-h)^2$ $x\text{-}h = a(y\text{-}k)^2$ Circles: $(x-h)^2 + (y-k)^2 = r^2$ Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Limits

KY.HS.C.1 (+) Understand limits. a. Apply limits to a variety of functions, including piecewise functions. b. (++) Prove that the limit of a function exists, based upon the definition of a limit. **MP.2, MP.3**	
KY.HS.C.2 (+) Demonstrate an understanding of limits by estimating and finding the limit of a function at a point graphically, numerically and algebraically. **MP.5, MP.8** KY.HS.C.3 (+) Apply properties and theorems of limits, including limits of indeterminate forms. **MP.2, MP.3**	Include analysis of limits in piecewise functions. Algebraic techniques include but are not limited to factoring, multiplying by the conjugate and finding the lowest common denominator. Include sums, differences, products, and quotients.
KY.HS.C.4 (+) Communicate understanding of limits using precise mathematical symbols and language. **MP.3, MP.6**	Use of limits to predict the function value for an undefined value (hole in the graph).
KY.HS.C.5 (+) Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **MP.2, MP.5**	

KY.HS.C.6 (+) Discuss the end behavior of functions; identify representative functions for each type of end behavior using precise mathematical symbols and language. **MP.2, MP.6**	$\lim_{x \to \infty} f(x) = 4$ implies a horizontal asymptote of y = 4 $\lim_{x \to \infty} f(x) = \infty$ implies right hand end behavior is positive infinity NOTE: odd functions result in end behavior similar to lines (opposite directions); even functions result in end behavior similar to parabolas (same direction)

Polar Coordinates and Equations

• Plot points in the polar coordinate system and generate multiple polar coordinates for the same rectangular point	
• Convert points and equations from rectangular to polar form and vice versa	
• Graph polar equations by plotting points and using symmetry, zeros and maximum r values as graphing aids	
• Recognize graphs and equations (lines, circles, lemniscates, limacons, rose curves) in polar form.	